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Equations of an Oblique Projectile Motion without Calculus

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Abstract: *This Manuscript involves the derivation of the equations of motion of a projectile round an oblique path. Using the three equations of motion in Physics, we derived the equation for the time to reach the maximum height from the first equation of motion in Physics, the time of flight equation from the second equation of motion in Physics, the Maximum height equation from the third equation of Motion in Physics and lastly the horizontal range equation from the knowledge of the definition of distance in Physics which is the product of the average velocity and the time taken during the course of the motion.*

Keywords: *sinθ, cosθ, eqn*

INTRODUCTION

The motion of an object is called two dimensional, if two of the three co-ordinates are required to specify the position of the object in space changes w.r.t time. In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun etc. Two special cases of motion in two dimensions are 1. Projectile motion 2. Circular motion
A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path. The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion. If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.
A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.
Example: (i) A bomb released from an aeroplane in level flight (ii) A bullet fired from a gun (iii) An arrow released from bow (iv) A Javelin thrown by an athlete.

Assumptions of Projectile Motion

1) There is no resistance due to air.
2) The effect due to curvature of earth is negligible.
3) The effect due to rotation of earth is negligible.
4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

Types of Projectile Motion

1. Oblique projectile motion
2. Horizontal projectile motion
3. Projectile motion on an inclined

Oblique Projectile

In projectile motion, horizontal component of velocity (u cosθ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

Horizontal Projectile

A body be projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

Projectile Motion on an Inclined Plane

The motion of a particle projected up with a speed u from an inclined plane which makes an angle α with the horizontal velocity of projection that makes an angle θ with the inclined plane is called Projectile Motion on an Inclined Plane (Kshetrapal, 2013).

To be a true projectile, an object must:

1. Have negligible lift and drag from the air.
   - A Frisbee would not be a projectile because of its lift.
   - A wad of paper thrown through the air at a fast speed is not a true projectile due to the amount of air friction or "drag".

2. Have no means of self propulsion.
   - A model rocket would not be a true projectile because it is self propelled.

3. Have a short trajectory compared to the size of the earth.
   - If the altitude reached by an object is great enough, 9.80 m/s^2 would not be constant.
   - If the distance an object travels is great enough, the rotation of the earth would have to be taken into account.

If an object is a true projectile:

1) The center of mass follows a parabolic trajectory.
2) The rest of the object rotates about its center of mass.
3) Gravity is the only force acting on it.[1]

In This research article we are primarily concerned about deriving the equations of projectile Motion in an oblique path.

Methodology

In this section we concerned about the derivation of the equations of projectile Motion in round an oblique path.

[1] Projectile Motion Notes Name, https://www.grantbulldogs.org

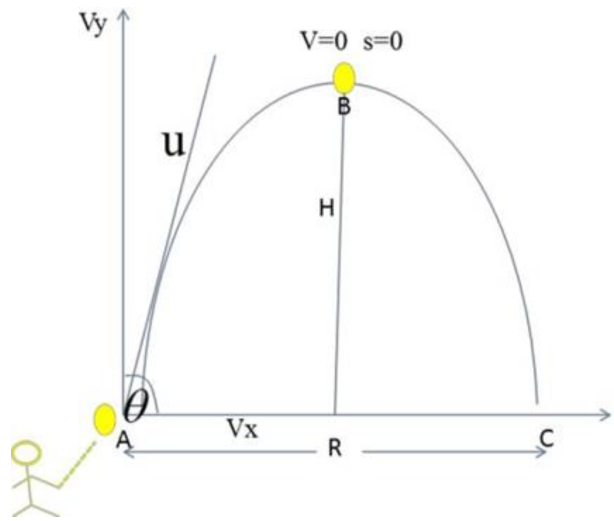

Figure 1: The Path described by a projectile

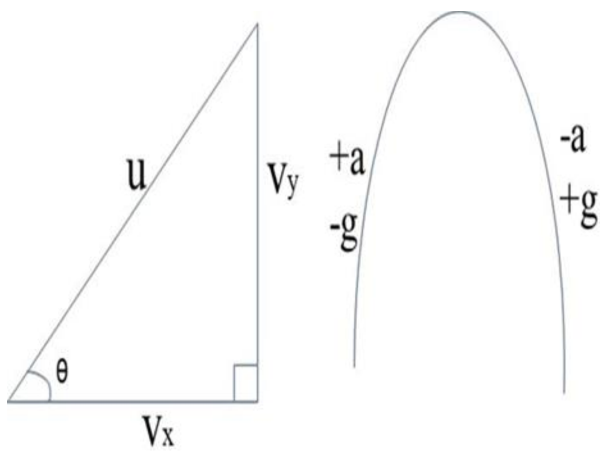

Figure 2: (Resolution of velocities and the path described by gravitational accelarations)

Given that a boy throws a stone toward an orange on a tree, and the path described by the motion of stone is from point A towards point B and then towards point C, as shown in Figure 1

From Figure 2 above we've

$$\sin\theta = \frac{v_y}{u}$$

$$\Rightarrow v_y = u\sin\theta \qquad (1)$$

$$\cos\theta = \frac{v_x}{u}$$

$$\Rightarrow v_x = u\cos\theta \qquad (2)$$

From the three equations of motion we've

$$v = u + at \qquad (3)$$

$$s = ut + \frac{1}{2}at^2 \qquad (4)$$

$$v^2 = u^2 + 2as \quad (5)$$

From Figure 2 we've

$$a = -g \quad (6)$$

Putting eqn (6) into eqn (3), (4), & (5) we've

$$v = u - gt \quad (7)$$

$$s = ut - \frac{1}{2}gt^2 \quad (8)$$

$$v^2 = u^2 - 2gs \quad (9)$$

Time to reach the maximum height

The time taken for the stone to reach the maximum height is the time in which the stone takes to travel from point A towards point B, Where the initial velocity of the stone to reach the maximum height is Vy since the stone is thrown vertically upwards.
From eqn (7) we've

$$v = v_y - gt \quad (10)$$

$$0 = v_y - gt \quad (11)$$

Substituting eqn (1) into eqn (11) we've

$$0 = u\sin\theta - gt \quad (12)$$

$$\Rightarrow gt = u\sin\theta$$

$$t = \frac{u\sin\theta}{g} \quad (13)$$

The equation above is the equation used to calculate the time taken (t) for the stone (projectile) to reach the maximum height at point B in Figure 1. above.

Time of Flight

The time taken for the stone to travel through point A through point B to point C is the time of flight (T).
From eqn (8) above we've

$$s = v_y t - \frac{1}{2}gt^2 \quad (14)$$

At maximum height $s = 0$

$$0 = v_y t - \frac{1}{2}gt^2 \quad (15)$$

$$v_y t = \frac{1}{2}gt^2 \quad (16)$$

$$2v_y = gT^2 \quad (17)$$

$$T = \frac{2v_y}{g} = \frac{2u\sin\theta}{g} \quad (18)$$

$$\Rightarrow T = \frac{2u\sin\theta}{g} \qquad (19)$$

Eqn (19) is the equation for time of flight.

The Maximum Height

From eqn (9) we've $u = v_y$, $s = H$

$$\Rightarrow v^2 = v_y{}^2 - 2gH \qquad (20)$$

At maximum height $v = 0$

$$\Rightarrow 0 = v_y{}^2 - 2gH \qquad (21)$$

$$\Rightarrow v_y{}^2 = 2gH \qquad (22)$$

$$H = \frac{v_y{}^2}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

$$H = \frac{u^2\sin^2\theta}{2g} \qquad (23)$$

Equation (23) is the equation for calculating the maximum height of a projectile (in this case a stone).

Range

Range is the distance between the initial point of the stone (projectile) i.e. point A before it is thrown vertically upwards to the point where the stone (projectile) fall downwards to the ground i.e. point C as shown in figure 1. above.

$$\Rightarrow Range = dis\tan ce\ from\ po\text{int}\ A\ to\ p\text{oint}\ C$$

Given that from the definition of velocity we've

$$v = \frac{s}{t} \Rightarrow s = vt \qquad (24)$$

Where S=R (range), V= V_x {since the velocity is horizontal}, t= time of flight since the stone (projectile) travels from point A to C

$\therefore eqn(24)\ becomes$

$$R = v_x T \qquad (25)$$

$$where\ T = \frac{2u\sin\theta}{g} \text{ (Time of flight)}$$

$$\Rightarrow R = v_x \times \frac{2u\sin\theta}{g} \qquad (26)$$

Putting eqn (2) into eqn (26) we've

$$\Rightarrow R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2.2\sin\theta\cos\theta}{g} \qquad (27)$$

From trigonometry we've

$$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= 2\sin\theta\cos\theta$$

On substitution into eqn (27) we've

$$R = \frac{u^2 \sin 2\theta}{g} \qquad (28)$$

Eqn (28) is the equation for calculating the horizontal range of a projectile.

Applications of Projectile

The principles of Projectile Can be applied in Military Vehicles for aiming accurately at target and it can also be applied in Sports such as Skating, Javelin to determine the Maximum height of an individual, Ranges etc (BLN Ndupu and PN Okeke, 2010; Hugh and Roger, 2012; Anyakoha, 2013).

Conclusions

From eqn (13), (19), (23), and (28) we have successfully derived the equations for time to reach the Maximum height of an oblique projectile motion, time of Flight, Maximum height equation and the equation for the Horizontal Range.

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