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An Investigation on the Thermodynamic Properties of Hot Nuclear Material in the Isentropic Condition

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Abstract: *Investigating the nuclear material system as a many-body system is considered as an important issue in theoretical physics. In the finite nuclear material, the nucleus has a large area that its density becomes near to zero by getting distance from the center. Even for heavier nuclei, only a small fraction of the nucleons are in the center that it is assumed that there, the nucleonic density is constant. But, for many theoretical types of research, if we assume that the density is uniform all around the nucleus, the work will be easier. For this reason, the concept of infinite nuclear material as an ideal system of nucleons with the uniform density was formed which is similar to a heavy nucleus center. This kind of system is appropriate for the test of the nucleon-nucleon interaction. In addition, by having an infinite system, we are not obligated to calculate the complexities due to the mass center movement. Usually, we neglect the Coulomb interaction between the nucleons. Therefore, if we want to study the real nucleus, we have to analytically enter the properties related to the electromagnetic interactions into the problem. The neutron star is the closest sample to this kind of system.*

Keywords: *Hot nuclear material, System parameters, Equation of state, Isentropic.*

INTRODUCTION

Explaining the nuclear powers has been one of the main objectives of scientists from the early days of nuclear physics genesis. The scientists knew that nuclear power is very strong since 1934 and its range is about 2fm, but they have no information about its source. Until in 1935, a young Japanese scientist with the name of Hideki Yukawa proposed that a new type of quantum can be the reason for this phenomenon. Yukawa assumed that the main factor of this powerful force is a new particle that is transferred in the nucleolus of the nucleus and has a mass between the masses of electron and proton. This idea was confirmed by the discovery of π-meson or pion in nuclear emulsions by Frank Powell and G. P. S. Occhialini in 1947.

The pion has a mass equal to 270 times of electron mass and exists in three forms of positive, negative and neutral electric charges.

In the interactions related to two protons or two neutrons, the neutral electric charge of it and in the interactions related to the one proton and neutron, the charged type of it are transferred. The range of nuclear power is related to the pion mass by the equation of $R = \frac{\hbar}{mc}$ and is calculated to be about 1.4 fm (Freire et al., arXiv: 0902.2891).

Besides the π-meson, there are mesons with higher masses that are involved in nuclear forces. The most prominent of them are 1) the η-meson with the mass of about 549 MeV, 2) ρ-meson with the mass of about 769

MeV and 3) ω-meson with the mass of about 783 MeV. Therefore, the equation related to the range, predicts the range of heavier mesons significantly less than the range related to the pion.
The properties of inter- nucleonic forces could be studied directly in the collision of two nucleons (dispersion) or indirectly by extracting them from the restrained systems properties, i. e. the nuclei. Some of these properties are:

1. This force is stronger than the Coulomb force in the short ranges because nuclear power can overcome the repulsive Coulomb forces of the protons in the nucleus (Kenneth S. Kranne, 1988).
2. The nuclear power strongly weakens in the long ranges that are about the atomic dimensions, i. e. 2.5 fm, and it could be neglected; in other words, the nuclear power is short range (Preston andBhaduri, 1975).
3. Some of the particles like electrons are not affected by nuclear power (Kenneth S. Kranne, 1988).
4. Form the important aspects of nuclear power is its freedom from the charge. The effective force between two nucleons is independent of whether the particles are two protons, two neutrons or one proton and one neutron.
5. The nucleon-nucleon force gets saturated. If each nucleon could have attracted each of the other nucleons, it was expected that the binding energy to be proportional to A^2 and all the nuclei have diameters equal to the nuclear power range. Both the predictions for the nuclei with A>4 are strongly opposing the experience. As a result, nuclear power gets the saturated meaning that a particle only attracts the other particles around itself. Other nucleons are either not affected or are repulsed.
6. In the short distances, these force is repulsive, meaning that the central part of the nucleus has a repulsive nucleus that keeps the nucleons in a specified distance. Usually, this part is considered as hardcore, but there is some potential that defines the softcore for the nucleus (Like Reid potential (Reid potential, 1968)).
7. The force between two nucleons depends on the orientation (being parallel or counter-parallel) of the nucleons spins. The force between two nucleons with parallel spins is stronger than the force between two nucleons with counter-parallel spins.
8. The nuclear forces include a tensor component (non-centralized) (Bernard L. Cohen, 1990). These properties are due to the four-pole electric torque of dutron that is non-zero and results that the consistency of orbital angular momentum to be violated. The angular momentum is one of the movement constants in the central force field.

The last properties emerge from the dutron quantum numbers and this fact that this article has only one restrained state.
Specifying the equation of state for hot nuclear material is one of the most important issues in nuclear physics. This hot and dense nuclear material is produced in the relativity heavy ion collisions, stars collapse, supernova explosions, and neutron stars, and so on. In the finite temperature, the equation of state for the hot nuclear material is obtained by determining the Helmholtz free energy.

Research Methodology:

In the Hamiltonian Field Theory (FT), it is expanded around its average field according to its fluctuations size. In this concept, the MFT is the zero order expansion of the Hamiltonian fluctuations, meaning that the MFT system does not have any fluctuations and according to this theory, we can replace all the interactions with an average field. Therefore, the multi-body problem is converted into an effective single-body problem. To continue the work with the average field method, we need an effective interaction that acts between two particles. Here, the M3Y-Reid-Elliott effective interaction is used that its radial part is completed by a zero range pseudo-potential. In order to that this interaction presents a better description of the nuclear material,

especially in determining the saturation condition, usually, it is multiplied by a density-dependent factor and call it the density-dependent M3Y interaction (DDM3Y). In the present work, in order for the M3Y interaction to present acceptable results for the finite temperature, in the non-zero temperature, instead of the density-dependent factor, the main interaction is named as the momentum-dependent coefficient (MDM3Y).
The two nucleonic effective interaction Hamiltonian related to momentum
In order to obtain the single-particle Hamiltonian that is proper for hot nuclear material, it is better to replace the density-dependent part with a momentum-dependent part. Therefore, the general form of two nucleonic momentum-dependent effective interaction could be written as follows:

$$v_{00}(r,p,\varepsilon) = t_{00}^{M3Y}(r,\varepsilon)h(p,\varepsilon)$$

$$v_{01}(r,p,\varepsilon) = t_{00}^{M3Y}(r,\varepsilon)h(p,\varepsilon)$$

In the above equations, the momentum-related part is defined as follows:

$$h(p,\varepsilon) = D(1 - B(\varepsilon)p^n)$$

The effective potential that each neutron and proton feel from the nucleus remainder is respectively equal to:

$$v_n(r,p,\varepsilon) = \frac{A}{2}[v_{00}(r,p,\varepsilon) + Xv_{01}(r,p,\varepsilon)]$$

$$/v_p(r,p,\varepsilon) = \frac{A}{2}[v_{00}(r,p,\varepsilon) - Xv_{01}(r,p,\varepsilon)$$

Therefore, the single-particle Hamiltonian of the system is equal to:

$$+\sum_{i=1}^{A}\frac{A}{2}[v_{00}(r,p,\varepsilon) \pm Xv_{01}(r,p,\varepsilon)] \quad H=\sum_{i=1}^{A}\frac{\hbar^2 k_i^2}{2m}$$

The + sign is related to the neutrons and – sign is related to the protons.
We assume the nucleons as the Fermi gas. Therefore, in the finite temperature, the nucleons distribution follows the Fermi–Dirac statistics.
In the method selected here, instead of using the total energy of a single-particle that includes the kinetic and potential energy, according to the Landau theory, we insert the potential energy in the average mass of the nucleons and consider it as the effective mass. Meaning that, instead of particles, we consider the pseudo-particles with the mass of$0 \leq \frac{m^*}{m} \leq 1$, in which, m is the average mass of nucleon. Therefore, the total single-particle energy will be as follows:

$$\varepsilon = \frac{P^2}{2m^*(\rho,T)} \quad (6-1\text{-}4)$$

In fact, m^* is considered as the variation parameter to minimize the free energy. Now, the only existing unknown parameter in the particle distribution functions is the $\mu_i(\rho,T)$ i. e. the chemical potential of the i^{th} particle. The chemical potential could be obtained by the particle number survival condition i. e. the following condition (Landau andLifshitz, 1989):

$$A = \sum_{i=1}^{A} n_i(k,\rho,T)$$

The above equation could be written for the neutrons and protons separately:

$$\rho_n = \frac{1}{\pi^2}\int_0^\infty n_n(k_n,\rho,T)k_n^2\, dk_n$$

$$\rho_p = \frac{1}{\pi^2}\int_0^\infty n_p\big(k_p,\rho,T\big)k_p^2\, dk_p$$

By numerical solving the above equations, the chemical potential for the neutrons and proton will be obtained. Now, by determination of the distribution function, we will calculate the kinetic energy, potential energy entropy and free energy of the particles.

Determination of Helmholtz free energy

The kinetic energy per unit nucleon of the hot asymmetric nuclear material

In order to calculate the Helmholtz free energy, at first, we calculate the expected value of the kinetic energy operator in the phase space:

$$< E_{Kin} >= \frac{2\,V}{(2\pi)^3}\frac{4\,\pi\hbar^2}{2m}\Big[\int_0^\infty k_n^4\, n_n(k_n,\rho,T)\, dk_n + \int_0^\infty k_p^4\, n_p\big(k_p,\rho,T\big)\, dk_p\Big]$$

Therefore, for the kinetic energy per unit nucleon, we will have:

$$\varepsilon_{kin} = \frac{<E_{Kin}>}{A} = \frac{\hbar^2}{2m\pi^2\,\rho}\Big[\int_0^\infty k_n^4\, n_n(k_n,\rho,T)\, dk_n + \int_0^\infty k_p^4\, n_p\big(k_p,\rho,T\big)\, dk_p\Big]$$

The potential energy per unit nucleon of the hot asymmetric nuclear material

The expected value of the single-particle potential operator dependent on the momentum in the phase space related to the proton and neutron is as follows:

$$< V >= \frac{2}{(2\pi)^3}\frac{A}{2}\int d^3r[t_{00}^{M3Y} + Xt_{01}^{M3Y}]\int_0^\infty h(p_n,\varepsilon)n_n(k_n,\rho,T)k_n^2\, dk_n + \frac{2}{(2\pi)^3}\frac{A}{2}\int d^3r[t_{00}^{M3Y} - Xt_{01}^{M3Y}]\int_0^\infty h\big(p_p,\varepsilon\big)n_p\big(k_p,\rho,T\big)k_p^2\, dk_p$$

And using that, the potential energy per unit nucleon of the hot asymmetric nuclear material will be defined as follows:

$$\varepsilon_{pot} = \frac{< V >}{A} = \frac{(J_{v00} + J_{v01}X)}{2\,\pi^2}\int_0^\infty D(1 - B(\varepsilon)\hbar^2k_n^2)n_n(k_n,\rho,T)k_n^2\, dk_n + \frac{(J_{v00} - J_{v01}X)}{2\,\pi^2}\int_0^\infty D(1 - B(\varepsilon)\hbar^2k_p^2)n_p\big(k_p,\rho,T\big)k_p^2\, dk_p$$

Results and Discussion

Thermodynamic properties of hot nuclear material

The pressure of the nuclear material

The pressure of the asymmetric nuclear material is one of the important thermodynamic quantities, especially in the determination of the saturation density of nuclear material and phase transition topic. The density that according to it, the pressure of the nuclear material will be zero, is the saturation density because, in this state, the system is in the equilibrium state.

For a hot nuclear material system with the free energy of "*f*", the pressure is equal to:

$$P(\rho, T) = \rho^2 \frac{\partial f(\rho, T)}{\partial \rho}$$

The calculation of the system pressure is very important for determining the equilibrium density of the system and also determining the phase transition.
As could be seen in Figure 1, for a constant temperature, the pressure of the asymmetric nuclear material in a constant density increases as the asymmetry increases. In such a way that, the minimum pressure is related to the totally symmetric nuclear material with X = 0 and the maximum pressure is related to the pure neutron material with X = 1. Of course, in big densities, this behavior inverses.
It could be observed in Figure 2 that, by keeping the asymmetry and density as constant parameters, the temperature increase results in the pressure increase.
In both of the figures, there is a range of density that the pressure is negative. These ranges are physically forbidden and no system would be found there. By having the system pressure, the saturation density of nuclear material i.e. the density that because of it the system is in the equilibrium state could be determined. The density that because of it, the system pressure would be zero is the saturation density because the pressure of a system that is in the equilibrium state is equal to zero.

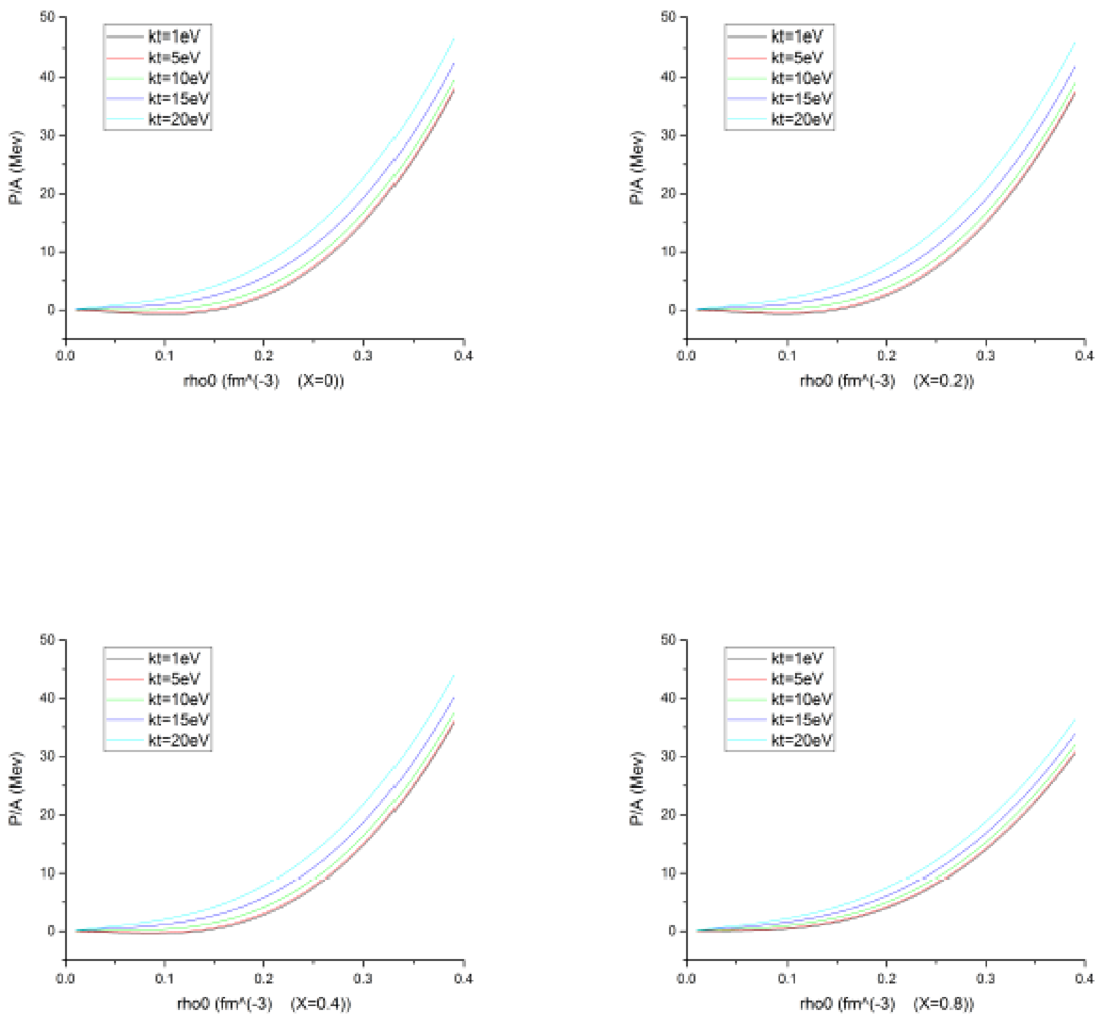

Figure 1: The diagram of pressure per unit nucleon of hot nuclear material versus density in isothermal condition.

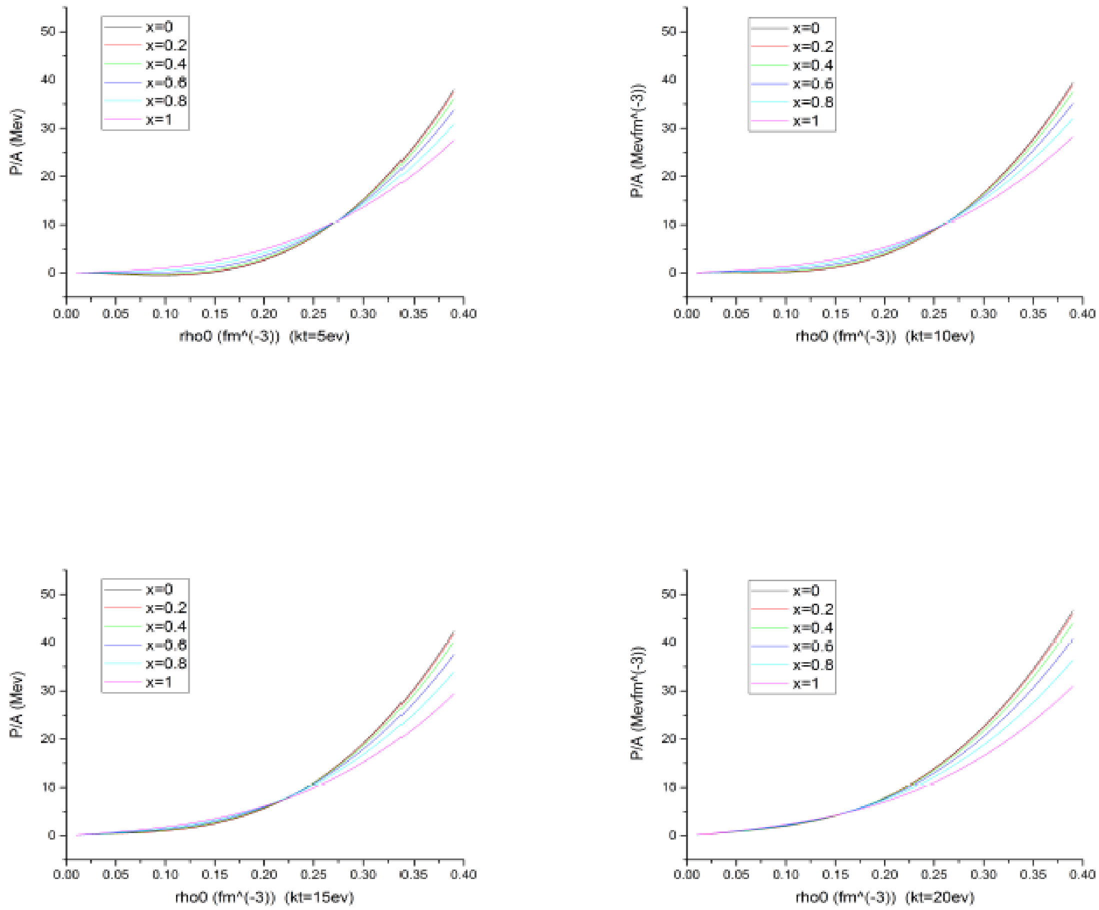

Figure 2: The diagram of pressure per unit nucleon of hot nuclear material versus density in fixed-asymmetry condition.

Now, we investigate free energy behavior in some entropies. In Figure 3, the free energy versus the nucleonic density for three different entropies in the temperature range of 1 eV to 20 eV has been plotted. As shown in the graph, the various asymmetric parameters are marked by various shapes. According to the graph, it could be understood that as the entropy increases, the minimum of the diagram decreases, for example, in the diagram number 1 – left side, the minimum for X = 0.3 is in the -10 MeV, but in the right side graph, it has been shifted towards the -15 MeV. In addition, with the increase of the entropy, the minimum happens in lower densities. On the other side, it could be understood from the graphs 1 and 2 that with the increase of asymmetry parameter, the minimum of the diagram gradually disappears and the system would not be stable anymore, as far as with the increase of entropy in the diagram number 3 – left side, no minimum could be observed.

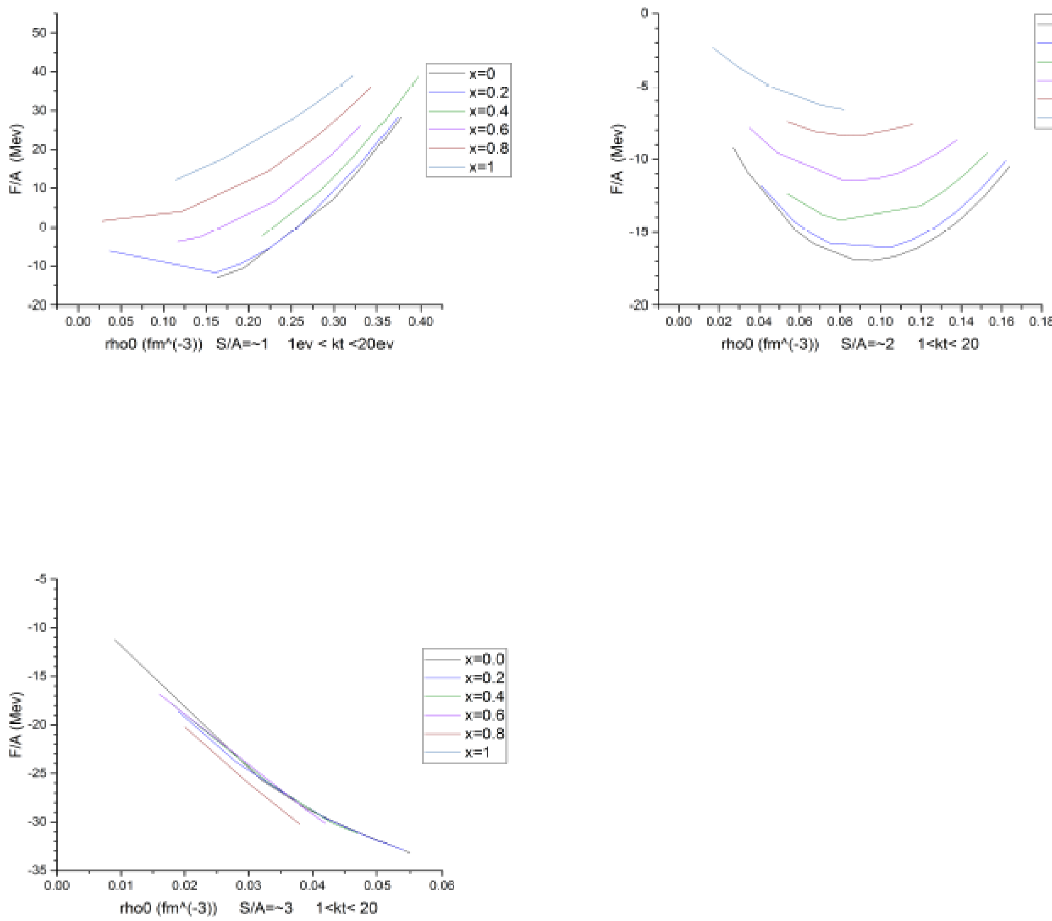

Figure 3: The diagram of free energy per unit nucleon in isentropic condition versus density in various asymmetry parameters.

In Figure 4, the Helmholtz free energy is plotted against the temperature in three various entropies. According to the graph, it could be understood that with the increase of entropy, the energy decreases. In $\frac{S}{A} = 1$ only for two initial asymmetries, the minimum is observable, and after that, the minimum is not seen. In$\frac{S}{A} = 2$, the minimum could be observed and with the increase of asymmetry, the minimum gets smaller till in the X = 1 the minimum is not seen. In $\frac{S}{A} = 3$, the behaviour of the system has changed and no minimum is observable.

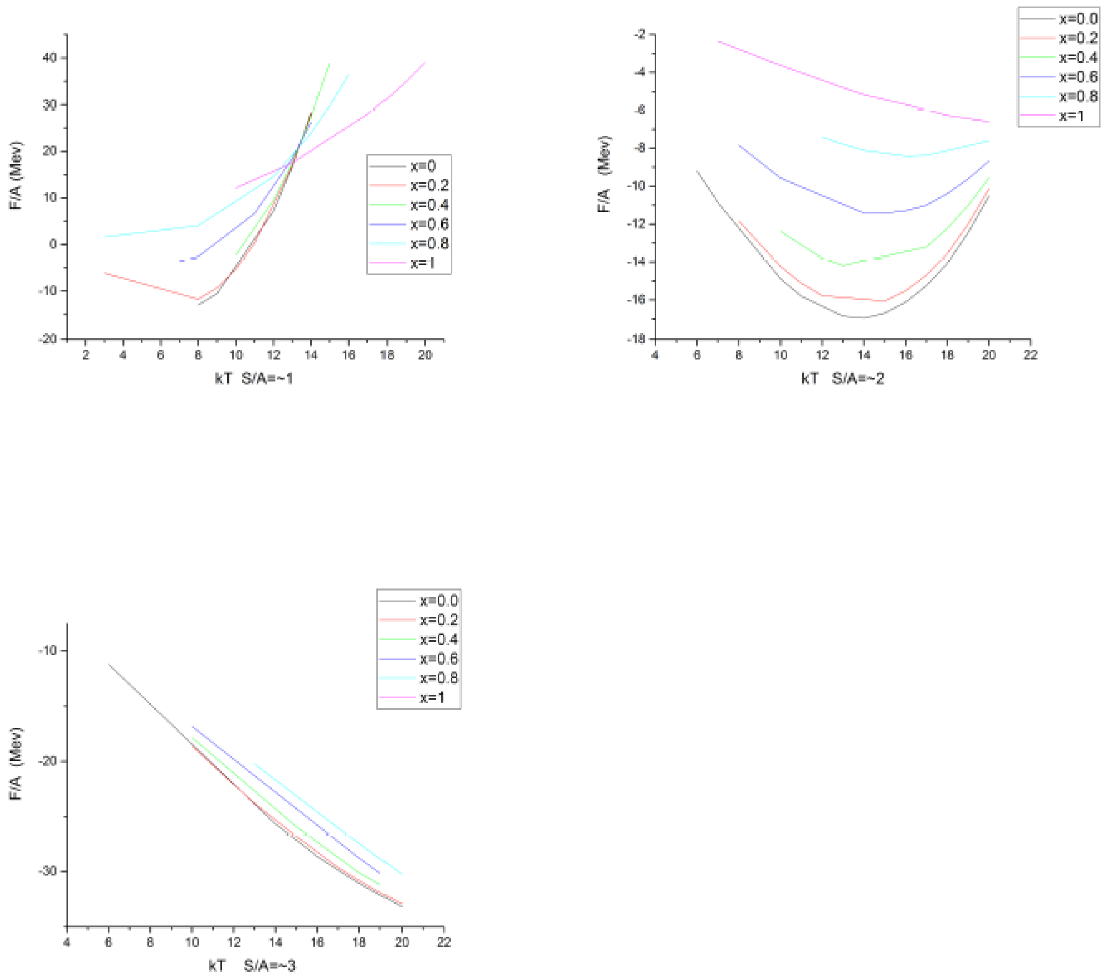

Figure 4: The diagram of free energy per unit nucleon in isentropic condition versus temperature in various asymmetry parameters.

In Figure 5, the pressure versus nucleonic density in three various densities has been plotted. According to the graph, it could be deduced that in $\frac{S}{A} = 1$, due to the various asymmetry parameter, the behavior is almost identical but with the increase of entropy, the graphs get separated until in the $\frac{S}{A} = 3$, the system behaviour changes and no minimum of pressure is observable.

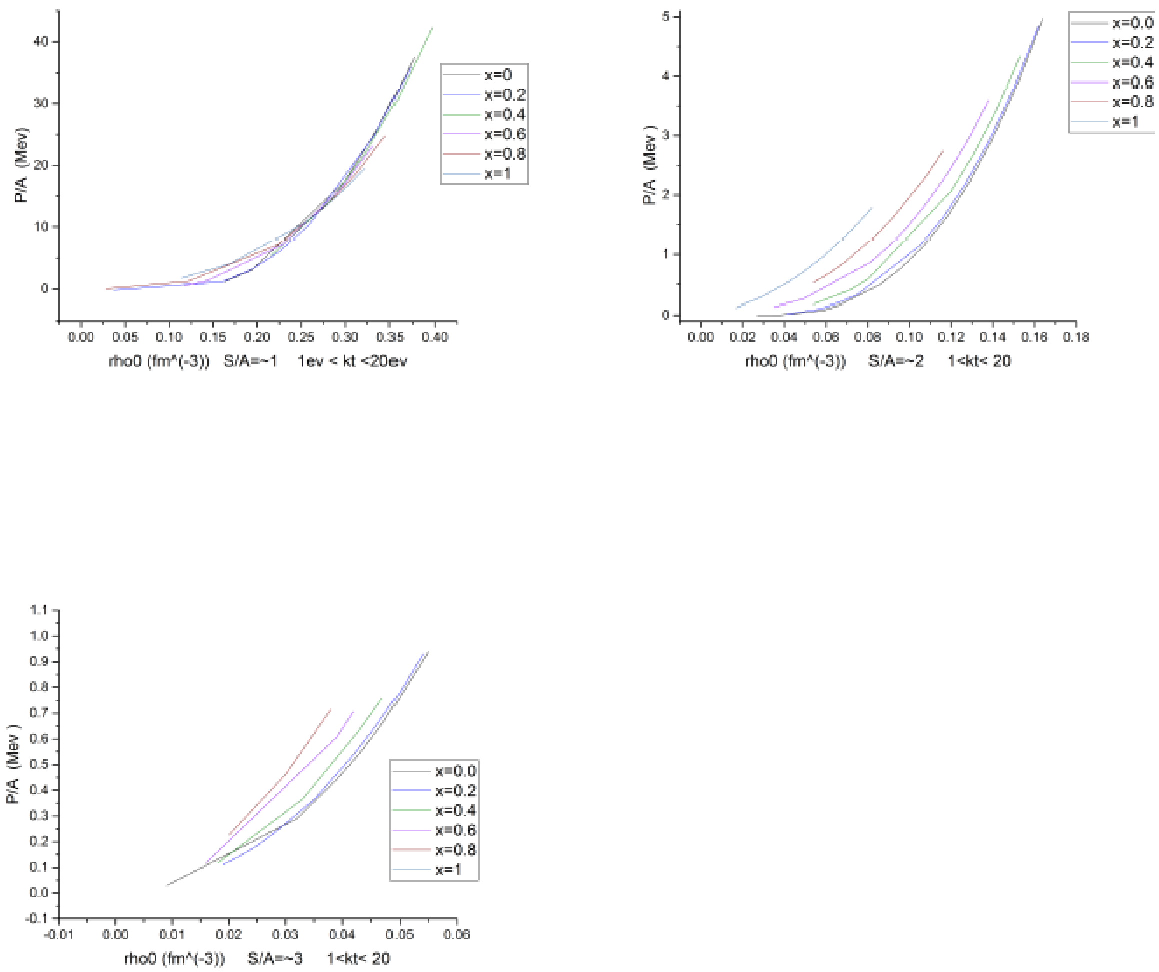

Figure 5: The diagram of pressure per unit nucleon in isentropic condition versus density in various asymmetry parameters.

In Figure 6, the pressure is plotted against the temperature in three various entropies. According to the graph, it could be seen that with the increase of entropy, the system pressure decreases.

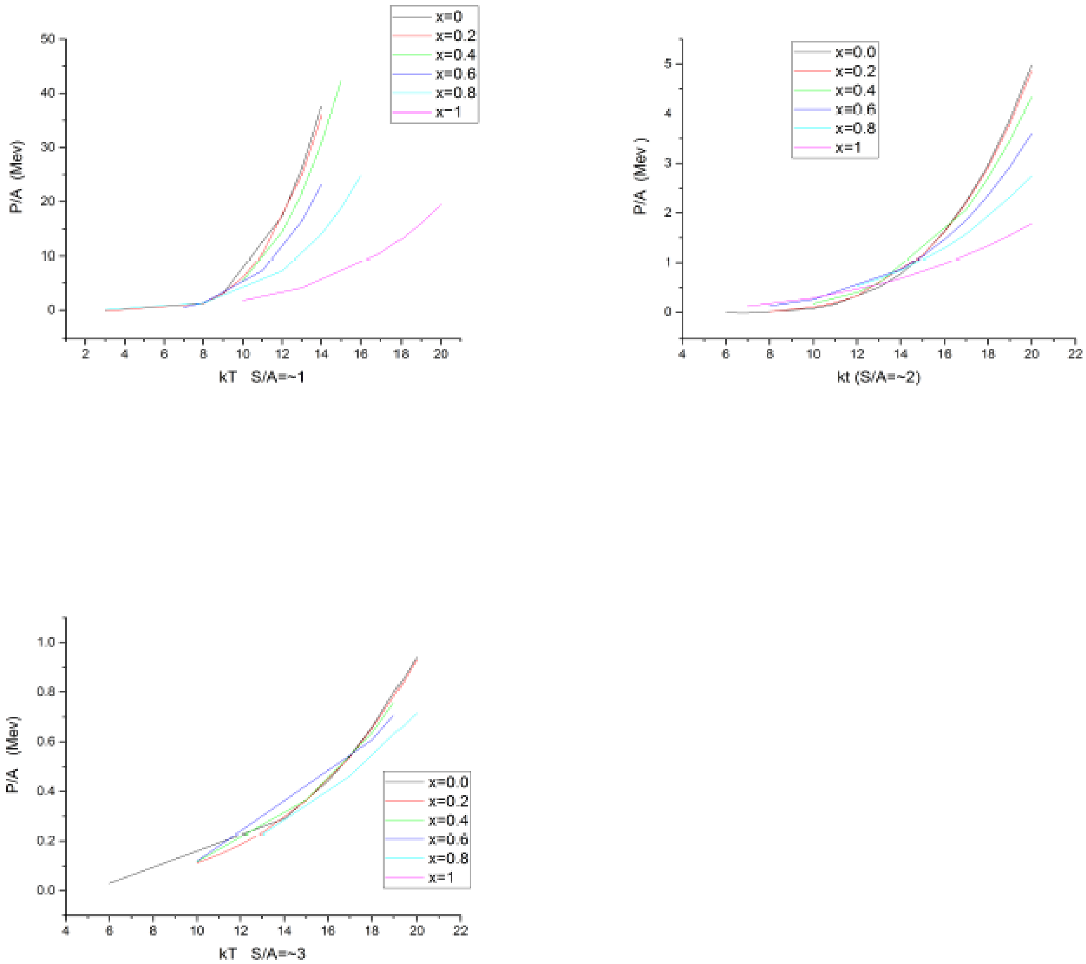

Figure 6: The diagram of pressure per unit nucleon in isentropic condition versus temperature in various asymmetry parameters.

In Figure 7, the chemical potential is plotted versus the nucleonic density for three various entropies. According to the graph, it could be understood that with the increase of entropy, the chemical potential decreases, the maximum chemical potential is related to the entropy per unit nucleon equal to 1, and minimum of it is related to the entropy per unit nucleon equal to 3.

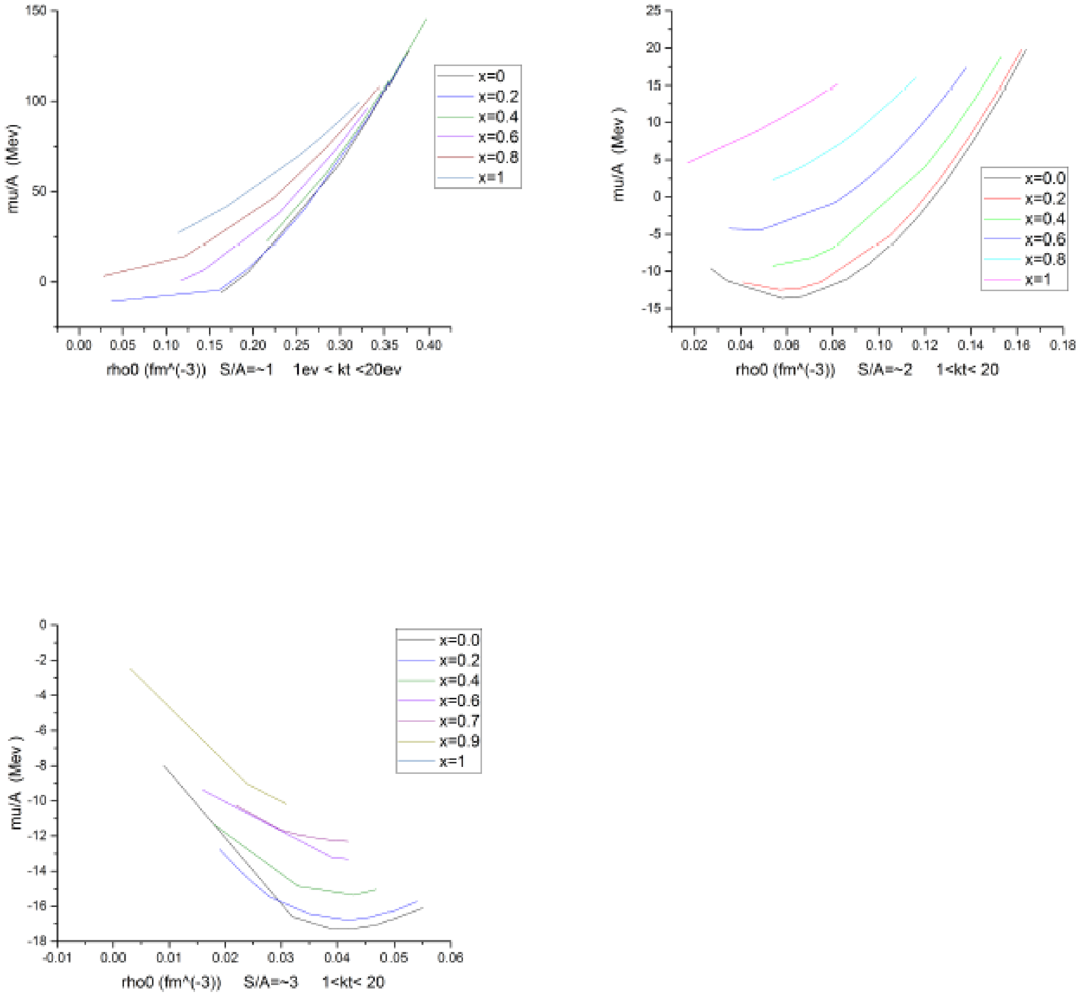

Figure 7-: The diagram of chemical potential per unit nucleon in isentropic condition versus nucleonic density.

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