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Presenting a Bi-Level Model and a New Hybrid Solution Algorithm for Inventory Routing Problem (IRP) Considering Motivational Designs

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Abstract: *This paper investigates a multi-product multi-period inventory routing problem in a two-echelon decentralized supply chain that consists of a supplier and a set of retailers. Here, it is assumed that the supplier and retailers make decisions independently. Hence, a nonlinear mixed integer bi-level programming model is proposed for the problem where the supplier and retailers are considered as leader and followers, respectively. In the proposed model, a discount scheme is applied as an incentive scheme to coordinate the supplier and retailers. In this paper, an efficient hybrid algorithm is developed to solve the proposed model. The proposed algorithm is designed based on Genetic and Ant Colony Algorithm.*

Keywords: *Inventory Routing Problem, Incentive Scheme, Bi-Level Programming.*

INTRODUCTION

IRP refers to coordination and integration of the decisions of two key components of the supply chain; i.e., transportation and inventory management. The goal of IRP is to determine an optimal policy regarding transportation and inventory management, so that their total cost could be minimized (Huang and Lin, 2010) and it is specified when, what products and through which route or means of delivery are sent to each retailer (Campbell et al., 1198). One of the basic assumptions in IRP is the complete coordination between the supplier and the retailers. Overall, there are two modes to reach coordination. The first mode is to ensure that all members of the supply chain are under unit ownership, and the second mode is that the members of the supply chain have independent ownership and coordination is done through coordination mechanisms.
One of the coordination mechanisms used for IRP independent-member mode is Vendor Managed Inventory (VMI) system. VMI is a term for inventory management systems where the supplier manages inventory-related activities (Disney and Towill, 2003). In other words, in VMI systems, retailers allow the supplier to control the timing and product delivery volume to the retailers. Instead of this freedom of action, the supplier ensures that the retailers will not face shortage of supply. In more traditional relationships between the supplier and retailers, where the retailers send product order requests to the supplier, due to the timing of retailers' orders, the performance may severely reduce, and consequently the costs of inventory and distribution increase sharply (Campbell and Savelsbergh, 2004). By using VMI system, the supplier can save

production and shipping costs and the retailers benefit from its implementation, as they do not allocate resources for controlling and managing inventory (Coelho, Cordeau and Laporte, 2012). Although implementation of VMI system is an effective way to reduce the costs of the supply chain, it is not easy to operate in practice, especially when the number and variety of retailers are high (Campbell and Savelsbergh, 2004), but the supplier can reach this goal by using IRP.

IRP has been investigated in several studies in the context of the coordination between suppliers and retailers through inventory management approach by the vendor. Andersson et al. (2010) presented a comprehensive overview and classification of previous studies on IRPs. Among the other studies reviewing the studies on IRP, one can refer to a paper by Moin and Salhi (2007) and Coelho et al. (2012). IRPs can be classified broadly according to the following criteria: finite or infinite plannings horizon (Anily and Federgruen, 1990; Archetti, Bertazzi and Laporte, 2007), different combinations of suppliers and retailers in the supply chain, such as single supplier multi-customer, single supplier single-customer or multi-supplier multi-customer (Archetti, Bertazzi and Laporte, 2007; Jaillet et al., 2002; Savelsbergh and Song, 2008). Moreover, they are the distribution of one or more product types during the planning period (Savelsbergh and Song, 2008; Huang and Lin, 2010), using homogenous or non-homogenous vehicles to send the products (Anily and Federgruen, 1990; Persson and Gothe-Lundgren, 2005), and the possible or definitive demand for products at the retail location (Kleywegt, Nori and Savelsbergh, 2002; Kleywegt, Nori and Savelsbergh, 2004; Abdelmaguid, Dessouky and Ordóñez, 2009). Additionally, IRPs are divided into IRPs by direct delivery (Barnes-Schuster and Bassok, 1997) or indirect delivery (multiple or continuous) in terms of the routing transportation items (Coelho, Cordeau and Laporte, 2012; Uggen, Fodstad and Nørstebø, 2011).

This paper examines the multi-period multi-product IRP in a bi-level supply chain consisting of a supplier and a set of retailers, where it is assumed that there is no possibility of coordinated and focused decision-making on transportation and inventory management like VMI. Thus, in this chain, each member is responsible for deciding on their own duties and tries to minimize their costs. Nevertheless, as already stated, using IRP requires full coordination between the supplier and the retailer. Thus, in this paper, an incentive scheme has been used to create coordination between transportation decisions of the supplier and the inventory management of the retailers with each other, and thus reducing the total cost of the supply chain.

Generally, incentive schemes are coordinated mechanisms that create incentives for supply chain members to behave in a decentralized supply chain, approximately or exactly like an integrated supply chain. Incentive schemes are designed with respect to the mutual goals of the members and to improve the relationship between them, after identifying the motivating cause (such as quantity, quality, price, etc.) (Tsay, 1999). The goal of the incentive schemes is to optimize the interests of the entire supply chain, minimize the cost of shortage and surplus inventory, and share fair risk among supply chain members (Arshinder, Kanda and Deshnukh, 2008). Among these incentive schemes, one can state quantity-discounting, repurchase, income sharing, etc.

An incentive scheme, a type of discount, is used in this paper to coordinate the decisions of the members of the supply chain regarding transportation and inventory management. The supplier to the retailers proposes this incentive scheme, where the supplier ensures that, if retailers make their inventory control decisions so that the supplier can efficiently utilize their transportation equipment, for the units purchased by the retailers, they reduce their costs to compensate for the coordinated decision-making by retailers. In addition, in this problem, it is assumed that:

- The products are delivered to retailers by a fleet of homogenous transportation devices with limited capacity and under multiple-transportation strategies to the retailers.
- The number of transportation means available to the supplier is limited, but if the supplier needs more transportation to deliver the products to retailers in a given period, they can reuse the existing vehicles by paying a fixed cost.
- The demand of the retailers is definitive and relatively small compared to the vehicle's capacity.

- The products sent to each retailer in each period must be delivered only by one of the means of delivery to the retailers
- The cost structure of the members of the supply chain is known and costs and prices are fixed.
- Inventory storage capacity by retailers is limited and shortage is unauthorized.

As in the discussed IRP, the supplier and the retailers are involved in decision-making, for the purpose of minimizing the costs of the supplier and the retailers, a Mixed-Integer Bi-Level programing model is presented to make a coordinated and decentralized decision on transportation and inventory management to cut supply chain costs as a whole to supply chain costs in an independent and decentralized decision-making mode. It has been shown that IRPs are among NP-Hard problems, so the above problem, which is a bi-level problem, is among NP-hard problems. Thus, an algorithm has been developed to solve this problem in large and reasonable time.
The structure of the paper: after the introduction in this section, in the next section, Mixed-Integer Bi-Level programing model for the proposed routing-inventory problem will be presented based on the assumptions made in the first section. In the third section, the algorithm solving method is expressed and the computational results of the solution of the model will be presented in the fourth section. Finally, in the fifth section, the conclusion is presented.

Mathematical Model

In the bi-level programming model, where the first-level decision maker is called the leader and the second-level decision maker follower, each decision maker tries to optimize his objective function regardless of the purpose of the other. However, the decision of each decision-maker affects the value of the objective function and the space for decision making of the other level. Thus, in bi-level programming, the first-level decision maker must select an answer as the optimal answer, which is also optimal for the second-level decision-maker.
In Mixed-Integer Bi-Level programing, IRP is presented according to the assumptions given in the introduction section. The symbols and signs used in the model are as follows:

A. Indices and sets:

i, j: index of the nodes (node 0 shows the supplier and nodes 1 to N show the retailers)
p: index of the products
t: index programming periods
N: number of retailers $N_0 = N \cup \{0\}$
P: types of products
T: The number of programming periods

B) Parameters:

h_{pjt} : the cost of maintaining a product type unit p at customer j in period t

A_j : Cost per order of customer j

d_{pjt} : the demand for product type p at the place of customer j in period t.

$I_{\max j}$: maximum storage capacity at the place of customer j

a_p : consumption coefficient of capacity of the retailers or capacity of the means of transport by type of product p

Q: maximum capacity of each transportation vehicle

c_{ij} : Transfer cost from node i to node j

V: Number of available vehicles
F: Fixed cost for reuse of each transportation vehicle in each period
C) Decision variables:
I_{pjt} : Inventory of product type p at the place of customer j at the end of period t

z_{jt} : zero and one variable: one if customer j sends an offer in period t, otherwise it is zero.

w_{pjt} : product type p ordered by customer j in period t

x_{ijt} : zero and one variable: One if seen immediately after seeing customer i and customer j in the period t

y_{jt} : the value of goods loaded before deliveries to customer j during the period t

K_t : the number of transportation vehicles required in addition to the current number (V) at the period t

δ_{pjt} : the value of discounts paid by the supplier to customer j during period t for the product type p

The IRP mathematical model according to equations (1) to (17) is as follows:

$$\min_{x_{ijt},K_t,\delta_{pjt},y_{jt}} \sum_{t\in T}\left[\sum_{i\in N_0}\sum_{j\in N_0|j\neq i} c_{ijt}.x_{ijt} + F.K_t + \sum_{j\in N}\sum_{p\in P}\delta_{pjt}.w_{pjt}\right] \tag{1}$$

$$\sum_{j\in N} x_{0jt} \leq V + K_t \qquad t\in T \tag{2}$$

$$\sum_{j\in N_0|j\neq i} x_{ijt} \leq 1 \qquad i\in N; t\in T \tag{3}$$

$$\sum_{i\in N_0|i\neq j} x_{ijt} = \sum_{i\in N_0|i\neq j} x_{jit} \qquad j\in N_0; t\in T \tag{4}$$

$$y_{jt} \leq y_{it} - \sum_{p\in P} a_p.w_{pit} + Q.(1-x_{ijt}) \tag{5}$$

$$j\in N_0; i\in N\,|\,i\neq j; t\in T$$

$$\sum_{p\in P} a_p.w_{pjt} \leq Q.\sum_{i\in N_0|i\neq j} x_{ijt} \qquad j\in N; t\in T \tag{6}$$

$$y_{jt} \leq Q \qquad j\in N_0; t\in T \tag{7}$$

$$x_{ijt} \in \{0,1\} \qquad i,j\in N_0\,|\,i\neq j; t\in T \tag{8}$$

$$K_t \in Z^+ \qquad t\in T \tag{9}$$

$$y_{jt} \geq 0 \qquad j\in N_0; t\in T \tag{10}$$

$$\delta_{pjt} \geq 0 \qquad p\in P; j\in N; t\in T \tag{11}$$

$$\min_{I_{pjt}, w_{pjt}, z_{jt}} \sum_{t\in T}\left[\sum_{j\in N}\sum_{p\in P} h_{pjt}.I_{pjt} + \sum_{j\in N} A_j.z_{jt} - \sum_{j\in N}\sum_{p\in P}\delta_{pjt}.w_{pjt}\right] \tag{12}$$

$$I_{pj(t-1)} + w_{pjt} - I_{pjt} = d_{pjt} \qquad p\in P; j\in N; t\in T \tag{13}$$

$$\sum_{p\in P} a_p.I_{pjt} \le I_{\max j} \qquad j\in N; t\in T \tag{14}$$

$$\sum_{p\in P} a_p.w_{pjt} \le Q.z_{jt} \qquad j\in N; t\in T \tag{15}$$

$$I_{pjt}, w_{pjt} \ge 0 \qquad p\in P; j\in N; t\in T \tag{16}$$

$$z_{jt} \in \{0,1\} \qquad j\in N; t\in T \tag{17}$$

The objective function (1) shows the supplier's objective function in the proposed model, which consists of two parts. The first part includes the costs of sending the product to the retailers and the costs of re-loading the means of transport, and the second part includes the discount that the provider pays for coordinating with the retailers. Constraint (2) shows the number of vehicles that the supplier needs in each period. In other words, this constraint shows the number of available vehicles (V) used in each supplier period and how many of these vehicles are reused. Constraints (3) and (4) ensure that products sent to each retailer in each period are delivered only by one means of delivery to the customer. Constraint (4) shows the continuity of the routes of the means of transport. Constraint (5) prevents the formation of sub-tours without the presence of the supplier. Constraint (6) shows the maximum volume of product that can be sent to each retailer in each period. Constraint (7) shows the capacity of the means of transportation; in other words, it states that the quantity of product loaded for customer j during period t must be smaller than the capacity of the means of transportation. Technical constraints on the variables of the supplier's decision are established through constraints (11-8). The objective function (12) shows the second level objective function of the proposed model; i.e., the objective functions of the retailer. This function includes the total cost of retailers' maintenance and ordering fees, minus the discount that retailers receive due to the coordination of the supplier. Constraint (13) represents the balance of inventory of retailers. It should be noted that I_{pj0} is assumed zero for every product and all retailers. Constraint (14) shows the maximum storage capacity at the retailer's place. Constraint (15) ensures that in each period, each retailer can maximize the capacity of a carrier for a variety of products. Constraints (16) and (17) show technical constraints on the model decision variables.

The Solution Approach

In this section, the hybrid algorithm developed for solving the model is discussed. The proposed algorithm consists of three steps:

Step 1: Calculating the cost and quantity of retailer's orders in independent decision making mode

In this stage, the genetic algorithm is used where the variables z_{jt} are considered as independent variables, and other variables under the control of retailers are calculated based on it. As a result, with some certain iteration, the quantity and cost of retailers are determined in the independent decision mode (Figure 1).

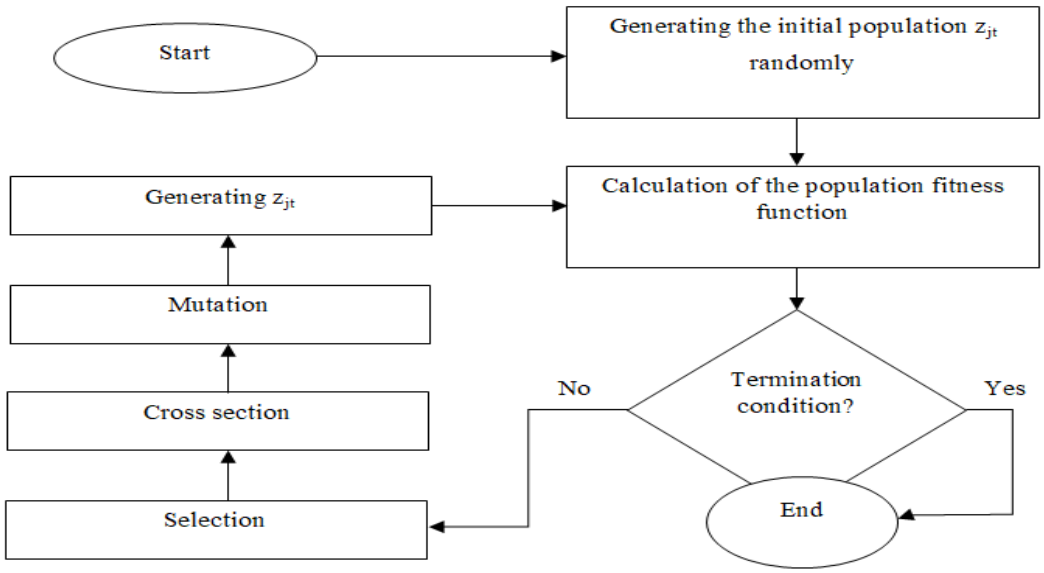

Figure 1: The algorithm of step 1

Step 2: Calculating the shipping cost of the supplier in independent decision mode

In this stage, to calculate the shipping cost of the supplier in each period, firstly, the retailers with orders are assigned to the vehicles. This is done according to Clark and Wright saving matrix method (Toth and Vigo, 2002). Then, for each vehicle, a transport route is determined using the ant colony algorithm (Figure 2).

Step 3: Calculating the costs of both levels in a coordinated state

This stage uses the data from the first and second stages to calculate the costs of both levels in a coordinated state, which has three stages (Figure 2):

Stage 1: Calculating shipping cost plus minimum discounts

At this stage, a genetic algorithm is used to minimize supplier costs, such as shipping costs and discounts. In the algorithm used in this stage, z_{jt} variables are considered as independent variables, and other variables under the control of retailers are the value of order and the amount of inventory is calculated according to it. Then, according to the obtained order values, retailers are assigned to vehicles according to the Clark and Wright saving matrix method. Later on, the shipping cost is calculated roughly according to the nearest neighbor method for each vehicle (Gutin, Yeo and Zverovich, 2002). In addition, the increase in retail costs is added to the supplier's costs relative to the cost of the stand-alone mode (first stage) as the minimum discount (in this case, the discount price of the retailers in both coordinated and independent terms).

Stage 2: Improving the answer of the previous stage

At this stage, using Ant Colony Algorithm, it is tried to improve the shipping cost of the best response from the first stage. Thus, at the end of these two stages, shipping, ordering and maintenance costs and minimum discount rates are determined. If, at the end of this stage, the reduction in supplier costs relative to independent decision making is greater than the increase in retail costs relative to independent decision making (minimum discount rate), the algorithm moves on to the next stage, or the costs of both levels are equal to the cost obtained in the independent mode.

Stage 3: The amount of discounts to customer j during period t for the type product p (δ_{pjt})

At this stage, δ_{pjt} value is first determined randomly and the total discount is calculated based on it. Then if the total discount obtained at this stage is not the same with the minimum discount, δ_{pjt} is modified by the following equation.

$$\delta_{pjt} = \frac{\textbf{minimum discount}}{\textbf{\textit{total discount}}} \times \delta_{pjt} \tag{18}$$

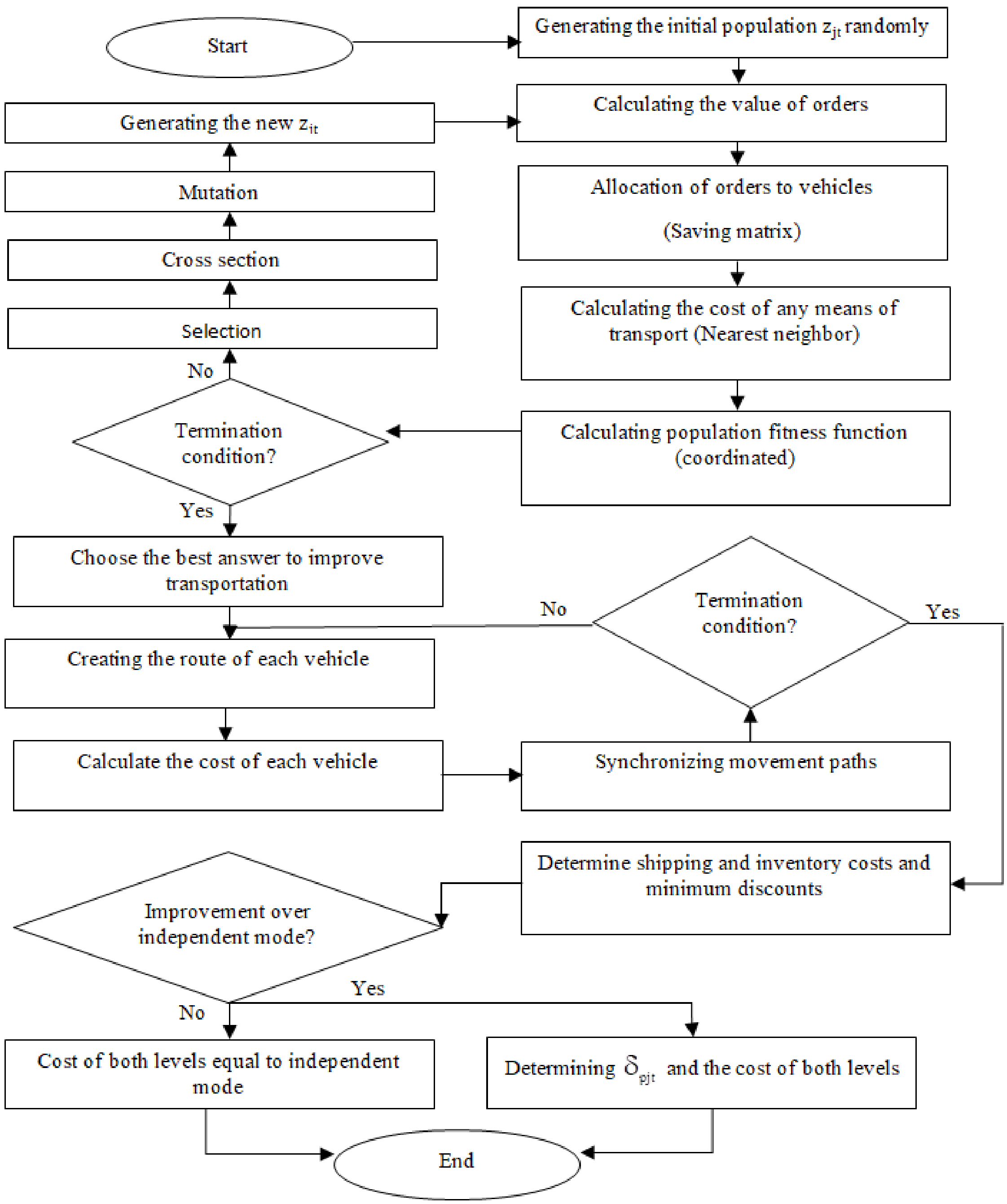

Figure 2: The algorithm of step 2 and step3

Numerical results

This section examines the numerical model and proposed algorithm to determine their performance. In doing so, 28 sample problems were produced at different dimensions. In these problems, generating the parameters is done according to the table below.

Table 1: Generating the parameters

Parameter name	Parameter value
c_{ij}	U[50,500]
d_{pjt}	U[0,50]
A_j	100
h_{pjt}	1
a_p	1
Q	{500,800,1200,1700}
V	{1,3,4,5}
$I_{max\,j}$	Q
F	20

The results from solving problems designed in independent and coherent decision-making conditions are shown in Table 2. As is seen from the results, co-ordinated decision making based on the bi-level model has ended in lower supplier costs and constant customer spending. The proposed model, on average, has resulted in a 23.03% reduction in supplier costs, which means that the supplier has given a discount on retailers to keep their costs constant and their total cost includes shipping costs and a discount on independent decision making in addition to reducing shipping costs. Thus, using the proposed model can lead to improved supply chain performance when customers tend to adopt inventory control decisions.

Table 2: The results of solving sample problems

Problems	Problem dimensions P×N×T	Supplier		Retailers	
		Independent	Coordinated	Independent	Coordinated
1	3 × 1 × 7	4850	2244	655	655
2	3 × 1 × 10	2100	1337	847	847
3	5 × 1 × 7	5138	2415	700	700
4	5 × 1 × 10	3240	1791	1000	1000
5	3 × 5 × 7	5624	4003	2894	2894
6	3 × 5 × 10	8231	5863	4108	4108
7	5 × 5 × 7	11008	6742	3376	3376
8	5 × 5 × 10	12703	8190	4619	4619
9	3 × 10 × 7	8515	6866	5777	5777
10	3 × 10 × 10	14333	11292	8593	8593
11	5 × 10 × 7	13168	8978	6826	6826
12	5 × 10 × 10	17739	14297	9494	9494
13	3 × 18 × 7	13981	10816	10836	10836
14	3 × 18 × 10	20026	17079	15258	15258
15	5 × 18 × 7	15873	12260	12104	12104
16	5 × 18 × 10	26650	22442	17518	17518

17	3 × 27 × 7	17250	15541	16318	16318
18	3 × 27 × 10	29518	25488	23527	23527
19	5 × 27 × 7	26593	22697	18495	18495
20	5 × 27 × 10	38174	32422	26340	26340
21	3 × 45 × 7	25436	23330	26668	26668
22	3 × 45 × 10	36328	32288	39069	39069
23	5 × 45 × 7	37208	29890	30967	30967
24	5 × 45 × 10	54250	45632	44367	44367
25	3 x 50 x 7	27389	25486	30666	30666
26	3 × 50 × 10	39260	34372	43435	43435
27	5 × 50 × 7	42700	36380	34423	34423
28	5 × 50 × 10	39651	33653	49334	49334

Conclusion

This paper proposed a Mixed-Integer Bi-Level programing model and a hybrid algorithm for IRP in a bi-level supply chain consisting of a supplier and a set of retailers to decide on inventory control and transportation in a coordinated way in which each member of the supply chain is responsible for his work. In this paper, it is assumed that a fleet of limited-capacity carriers ships several types of products from the supplier with a limited capacity to a set of retailers with multiple delivery strategies. In addition, for each retailer in each period, it must be shipped at maximum by a vehicle and once. The inventory maintenance capacity at the customer's site is finite and the shortage is unauthorized. Furthermore, the supplier can use the means of transportation if necessary. In the proposed model, the supplier is considered as the leader on the first level, and customers are ranked as followers in the second level, whose goal is to minimize their costs. The performance of the proposed model was evaluated by solving sample problems by the combined algorithm presented for the model and the results from solving sample problems represent the improvement of supply chain performance. Thus, using this model in a situation where members of the supply chain make decisions on inventory management and logistics in a decentralized way, the members of the supply chain will see the effect of their decisions on other members' decisions. Moreover, the performance of supply chain will improve compared to the situation where decisions are completely independent because of coordinated decision on inventory management and transportation.

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