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Optimization of Zayande Rood Dam Water by Using Stochastic Dynamic Programming

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Abstract: *One of the problems that threatens the earth planet is the problem of water shortage and circumstance of it's allocation. In the recent decades, according to increasing demand for drinking water, industry and agriculture as well as the reduction of groundwater resources, this problem has been intensified. This paper presents a new and fundamental method for the allocation of water in areas faced water shortages using stochastsic dynamic programming. In this study, demands, the amount of precipitation and input water to dam are considered probabilistic. We developed a dynamic programming based model with objective to determine the quantities that have to be allocated at each period in order to minimize the total cost. Finally, a numerical example is worked out to illustrate how the model is applied.*

Keywords: *Dynamic Programming, Dam Inventory, Demand, Holding Cost, Backlog Cost, Conduction Cost*

INTRODUCTION

Literature review

The increasing demand for water consumption and decreasing groundwater have caused a crisis in water supply. In the recent years, different studies have been done for water resources allocation around the world. The researches consist of two main approaches. Firstly, the dynamic programming based methods are common classical approaches. In this kind of problems, the researchers have simplified the model by using mathematical methods and probability rules. Cervellera et al. (2006) presented a method for decreasing the dimensions of water allocation problem and removing some of the states; In this study, state variables have been considered continuous, Objective functions and constraints have been considered nonlinear and finally the solutions have been approximated. Heidari and Jamshidi (1977) designed a metaheuristic method to control and optimize allocation of Khuzestan water resources called discrete differential dynamic programming (DDDP).The technique makes use of an initial nominal state trajectory for each state variable and forms corridors around the trajectories. Ben Alaya et al. (2003) have proposed an optimum solution to manage and improve water resources for arid condition in India. The water storage is used for irrigation. The determination of an optimal rule is based on two opposite objectives: the satisfaction of irrigation water demand and the safeguard of a minimal storage in dam. By considering different weights for these objectives, the stochastic dynamic programming has led to different optimal rules for the water resources management of the Nebhana dam. Piantadosi et al. (2008) presented a new method to determine a way for management of urban storm-water by using stochastic dynamic programming. They minimized conditional value-at-risk (CVaR) by using this method. They considered random inflow and constant demand. Luo et al. (2007) proposed a water resources system planning by using interval stochastic dynamic programming. They used the uncertainty and dynamic properties for obtaining reasonable solutions. They also analyzed the results and solutions for determining the main factors that affect the system's performance. Secondly, they have searched optimum solutions by fuzzy logic or multi objective concepts in some cases. In a study, Hall and East (1994) used genetic algorithm and dynamic programming for solving a multi reservoirs problem with objective to maximize profit of power generation and supply irrigation water. They reported predominance of GA to dynamic programming. Reddy and Kumar (2007) used GA,PSO and EMPSO for extracting utilization policies in multipurpose reservoirs that their objective was maximizing power generation and minimizing irrigation water shortage. Borhani Darian and Moradian (2010) decided to optimize utilization of multi-reservoirs systems. In this study, the utilization of multi-reservoirs has been accomplished by GA and ACO. The results indicated that as decision variables increase, ACO faces decreasing of optimality as well as increasing of calculation time. Zarghami and Hajikazemian (2013) have proposed a new optimization method based particle swarm optimization (PSOMS) that it's application was for water resources management in Tabriz (Iran). The objective functions were minimizing the cost, maximizing the water supply and minimizing the environmental hazards. Also the constraints were physical limits such as pipeline capacity, ground water, the demand and the impact of conservation tools. According to uncertainty of parameters, they are modeled by fuzzy logic and the problem is solved by proposal algorithm. Finally, the diversity of solutions is checked on the basis of an indicator of the distances between different solutions which show the efficiency of the PSOMS algorithm is preferred to the genetic algorithm. Chao-Chung Yang et al. (2009) integrated multi-objective genetic algorithm (MOGA) and constrained differential dynamic programming (CDDP) for supplying of surface and subsurface water. They simulated a supply system according to fixed costs, time dependent costs and financial constraints.

Research area situation

Zayande Rood is the biggest river of Iran plateau that originates from Zagros mountains and terminates to Gavkhuni swamp. The area of this river is about 41500 km^2 and it's length about 350 km. Zayande Rood dam has been constructed on this river that it's content is about 1460000000 m^3. Isfahan is the biggest consumer of Zayande Rood dam water in Iran. The water of Zayande Rood is allocated to 3 parts: agriculture, drinking water and industry. Regional water authority have encountered an critical problem for allocation water according to increasing of demand, decreasing of rainfall and transferring of some water to adjacent state (Yazd) in recent years.

Model Formulation

In this section we have formulated a model to describe the problem and to optimize it using stochastic dynamic programming. Before presenting the model, mathematical formulation and it's algorithm, the characteristics of dynamic programming should be presented.

Identifying states and stages in the model

The characteristics that a problem should possess to be able to be formulated as a dynamic programming problem are given as:

The problem can be divided into stages, with a policy decision required at each stage. In our model the different time periods in a finite time horizon are the stages. According to increasing water demand and water shortage during June to August, the first week of June till the last week of August are considered as stages of programming and at the beginning of these time periods we have to make a policy decision i.e. the quantity to be allocated to each area at that time period or stage. Each stage has a number of states associated with the beginning of that stage. In our model the states are the quantities of dam inventory at each stage. So the state variable is defined by the dam inventory in the beginning of each stage:

$$S_k = n_k$$

Where n_k is the quantity of dam inventory in the beginning at kth stage

The effect of the policy decision at each stage is depicted in the transformation of the current state to a state associated with the beginning of the next stage. In our model the policy decision (total of quantities to be allocated) at each stage will determine the state (size of inventory) of the next stage. The solution procedure is designed to find the optimal policy for the overall problem i.e. a prescription of optimal policy decision at each stage for each of the possible states. In our model we have to find the number of units to be allocated to each area at each stage to find overall optimal policy.

Given the current state, an optimal policy for remaining stages is independent of policy decisions adopted in previous stages. Therefore, the optimal immediate decision depends on only the current state and not on how you got there. This is the principle of optimality for dynamic programming. A recursive relationship that identifies the optimal policy for stage n given the optimal policy for stage n+1 is available. When we use this recursive relationship the solution procedure starts at the end and moves backward stage by stage each time finding the optimal policy for that stage until it finds the optimal policy starting at the initial stage. This optimal policy immediately yields an optimal solution for the entire problem.

Mathematical method

The model used here is integer linear programming (ILP). Before stating this model, we first introduce the following notation:

N: Total periods of allocation planning

k: Stage index

n_k: Dam inventory in the beginning of kth stage

C_h: Holding cost per unit of an inventory dam

C_a: Conduction cost per unit of inventory dam for agriculture

C_c: Conduction cost per unit of inventory dam for drinking water

C_i: Conduction cost per unit of inventory dam for industry

C_{ba}: Shortage cost for per unit of inventory dam for agriculture

C_{bc}: Shortage cost for per unit of inventory dam for drinking water

C_{bi}: Shortage cost for per unit of inventory dam for industry

a_k: Quantity of allocated water for agriculture at the kth stage

c_k: Quantity of allocated water for drinking at the kth stage

i_k: Quantity of allocated water for industry at the kth stage

r_j: jth possible state of arrival water to dam

u_l: lth possible state of water harvesting at the top of river

$d_{a,e}$: eth possible state for demand of agriculture

$d_{c,f}$: fth possible state for demand of drinking water

$d_{i,g}$: gth possible state for demand of industry

p_j: Probability of jth possible state of arrival water to dam

p_{u_l}: Probability of lth possible state of water harvesting at the top of river

p_e: Probability of eth possible state for demand of agriculture

p_f: Probability of fth possible state for demand of drinking water

p_g: Probability of gth possible state for demand of industry

α_n: Minimum quantity of dam inventory
M_n: Maximum capacity of dam inventory during planning horizon
α_a: Minimum quantity that should be allocated to agriculture
α_c: Minimum quantity that should be allocated to drinking water
α_i: Minimum quantity that should be allocated to industry
S_k: State variable at the kth stage
x_k: Decision variable at the kth stage

$C_{k,j,l}(S_k, x_k)$: Created cost subject to state variable and decision variable for jth possible state of arrival water to dam and lth possible state of water harvesting at the top of river at the kth stage

(1a) $Min\,[C_k(S_k, x_k) + f^*_{k+1}(S_{k+1})]$
Subject to

(1b) $f^*_k(S_k) = Min \sum_{m=1}^{N} \sum_{j=1}^{J} \sum_{l=1}^{L} [C_{m,j,l}(S_m, x_m)]\, p_j p_{u_l}$

$\forall\, k = 1,2,\dots,N \qquad \forall\, j = 1,2,\dots,J \qquad \forall\, l = 1,2,\dots,L$

(1c) $C_{k,j,l}(S_k, x_k) = C_{k,j,l}(n_k, a_k, c_k, i_k) = C_h(n_k + r_j - a_k - c_k - i_k - u_l) + C_a \times a_k + C_c \times c_k + C_i \times i_k + C_{ba} \times \sum_{e:d_{a,e}>a_k} p_e \times (d_{a,e} - a_k) + C_{bc} \times \sum_{f:d_{c,f}>c_k} p_f \times (d_{c,f} - c_k) + C_{bi} \times \sum_{g:d_{i,g}>i_k} p_g (d_{i,g} - i_k)$

(1d) $n_{k+1} = n_k + r_j - u_l - a_k - c_k - i_k$

$\forall\, k = 1,2,\dots,N \qquad \forall\, j = 1,2,\dots,J \qquad \forall\, l = 1,2,\dots,L$

(1e) $\alpha_n \le n_k + r_j - u_l - a_k - c_k - i_k \le M_n$

$\forall\, k = 1,2,\dots,N \qquad \forall\, j = 1,2,\dots,J \qquad \forall\, l = 1,2,\dots,L$

(1f) $\alpha_a \le a_k \le Max\,\{d_{a,e}\} \qquad \forall\, k = 1,2,\dots,N$

(1g) $\alpha_c \le c_k \le Max\,\{d_{c,f}\} \qquad \forall\, k = 1,2,\dots,N$

(1h) $\alpha_i \le i_k \le Max\,\{d_{i,g}\} \qquad \forall\, k = 1,2,\dots,N$

a_k, c_k, i_k *should be integer*

(1a) is objective function that minimizes total costs to optimize the model from first stage to kth stage. (1b) searches optimum state for each stage k by using expected cost. (1c) calculates cost subject to state variable and decision variable for jth possible state of arrival water to dam and lth possible state of water harvesting at the top of river at the kth stage. (1d) is a balance equation that assures inventory balance at the kth stage according to arrival water to dam, water harvesting at the top of river and summation of water quantity allocated to regions. Constraint (1e) indicate limitation for allocated water according to minimum quantity of dam inventory and maximum capacity of dam inventory during planning horizon. Constraints (1f),(1g),(1h) indicate limitation for allocated water according to minimum quantity that should be allocated and demand maximum in agriculture, city and industry respectively.

The proposed method

The presented model in previous section indicates a total scheme from system and inner relationships and shows only effective parameters and variables. Therefore a comprehensive algorithm should be presented to analyze and obtain optimum solutions. The following steps are described:

Step1: Estimate effective parameters on the model

Step2: Check out different allocation states for triple regions (a,c,i).

Step 3: Check out demand and allocation quantity conditions ; If all of the conditions are being established go to step 4, else return to step2.

Step4: Calculate transition probability matrix assuming distribution of water harvesting probability distribution (u) and arrival water probability distribution (r). $P_{m,n,a,c,i}$

Step5: Calculate objective function (cost) for each of the transition states. $C_{m,n,a,c,i}$

Step6: Calculate expected cost according to transition probability matrix ($P_{m,n,a,c,i}$) and transition cost ($C_{m,n,a,c,i}$).

(2a) $$\mathrm{E}(C_{m,a,c,i}) = \sum_{n:\min\{n\}}^{n:\max\{n\}} C_{m,n,a,c,i} \times P_{m,n,a,c,i}$$

Step7: Calculate $\sum_{n:\min\{n\}}^{n:\max\{n\}} p_{m,n,a,a,c,i} \times f_{N+1}^{*}(m)$ for each of the allocation states.

Step8: Search minimum cost for each state (m) as the optimum allocation state at the stage N.

(2b) $$\forall m: f_N^{*}(m) = Min[E(C_{m,a,c,i} + \sum_{n:\min\{n\}}^{n:\max\{n\}} p_{m,n,a,c,i} \times f_{N+1}^{*}(m)]$$

Step9: Check out stop condition; If the condition is being established the algorithm will finish, else return to previous stage and go to step7.

Results and Discussion

In this section, the results are expressed and they are discussed. Firstly, a numerical example is worked out and discussion is extended later.

Numerical example

In this section, we bring a numerical example to illustrate how the model works and algorithm is followed. As pointed, the first week of June to the last week of August are considered as stages. The solving policy is considered backward i.e. the first stage is considered the last week of August (16^{th} week) and the last stage is considered the first week of June (1th week). This example is a small scale from the main model and represent a comprehensive function of model and algorithm merely. Therefore, variables and parameters are rounded. The results of step1 that consists of effective parameters, has been presented in tables 1-7.

Table 1: Probability distribution of arrival water

r_j	15	16
p_j	0.4	0.6

Table 2: Probability distribution of water harvesting at the top of river

a+c+i	12	13	14	15
$p_{u_{l=2}}$	0.7	0.6	0.4	0.3
$p_{u_{l=3}}$	0.3	0.4	0.6	0.7

Table 3: Probability distribution of demand in agriculture

$d_{a,e}$	7	8
p_e	0.5	0.5

Table 4: Probability distribution of demand in city

$d_{c,f}$	4	5
p_f	0.6	0.4

Table 5: Probability distribution of demand in industry

$d_{i,g}$	1	2
p_g	0.2	0.8

Table 6: Unit costs

C_a	C_c	C_i	C_{ba}	C_{bc}	C_{bi}	C_h
100	600	200	1000	600000	500000	100

Table 7: Other parameters

α_n	M_n	α_a	α_c	α_i
1	4	7	4	1

After estimating the parameters, different allocations for triple parts (step2) are assessed by assuming allocation condition and demand condition (step3). After step2 and step3, transition probability matrix is calculated according to water probability distributions of water harvesting, arrival water and allocation that. The results are shown in table 8. The domain of states has been assumed fixed because simplicity of solving.

Table 8: Transition probability matrix, $P_{m,n,a,c,i}$

m	a	c	i	$p_{m,1,a,c,i}$	$p_{m,2,a,c,i}$	$p_{m,3,a,c,i}$	$p_{m,4,a,c,i}$
1	7	4	1	0.12	0.46	0.42	0
2	7	4	1	0	0.12	0.46	0.42
2	8	4	1	0.16	0.48	0.36	0
2	7	5	1	0.16	0.48	0.36	0
2	7	4	2	0.16	0.48	0.36	0
3	8	4	1	0	0.16	0.48	0.36
3	7	5	1	0	0.16	0.48	0.36
3	7	4	2	0	0.16	0.48	0.36
3	7	5	2	0.24	0.52	0.24	0
3	8	4	2	0.24	0.52	0.24	0
3	8	5	1	0.24	0.52	0.24	0
4	7	5	2	0	0.24	0.52	0.24
4	8	4	2	0	0.24	0.52	0.24
4	8	5	1	0	0.24	0.52	0.24
4	8	5	2	0.28	0.54	0.18	0

Next step is calculating created cost to system or same objective function (1c) and expected cost for each allocation consecutively for each allocation. This results are shown in table9. The mentioned table is also 16th stage of planning. Since $f_{16}^{*}(\text{m})$ is equal to zero, the results of 7th step is zero. Therefore, algorithm goes to step8 and optimum solutions for 16th stage is searched here that the results are shown in table10. After searching optimum solutions, algorithm goes to step9, stop condition is checked out and algorithm proceeds 1 stage according to not being established stop condition and algorithm returns to step7 again. Now, the calculations are done by assuming transition probability matrix and optimum values in the previous stage. These calculations and results are expressed in table11.

Table 9: Expected cost for different allocations

m	a	c	i	$C_{m,1,a,c,i}$	$C_{m,2,a,c,i}$	$C_{m,3,a,c,i}$	$C_{m,4,a,c,i}$	$E(C_{m,a,c,i})$
1	7	4	1	643900	644000	644100	0	644030
2	7	4	1	0	644000	644100	644200	644130
2	8	4	1	643500	643600	643700	0	643620
2	7	5	1	404500	404600	404700	0	404620
2	7	4	2	244100	244200	244300	0	244220
3	8	4	1	0	643600	643700	643800	643720
3	7	5	1	0	404600	404700	404800	404720
3	7	4	2	0	244200	244300	244400	244320
3	7	5	2	4700	4800	4900	0	4800
3	8	4	2	243700	243800	243900	0	243800
3	8	5	1	404100	404200	404300	0	404200
4	7	5	2	0	4800	4900	5000	4900
4	8	4	2	0	243800	243900	244000	243900
4	8	5	1	0	404200	404300	404400	404300
4	8	5	2	4300	4400	4500	0	4390

Table 10: Optimum allocations for 16th stage

m	a	c	i
1	7	4	1
2	7	4	2
3	7	5	2
4	8	5	2

Table 11: The results of calculations for different allocations at the 15th stage

m	a	c	i	$f_{16}^{*}(1)$	$f_{16}^{*}(2)$	$f_{16}^{*}(3)$	$f_{16}^{*}(4)$	$\sum_{m=1}^{m=4} p_{m,n,a,c,i} \times f_{16}^{*}(m)$	$E(C_{m,a,c,i})$	Sum
1	7	4	1	644030	244220	4800	4390	191640.8	644030	835670.4
2	7	4	1	644030	244220	4800	4390	33358.2	644130	677488.2
2	8	4	1	644030	244220	4800	4390	221998.4	643620	865618.4
2	7	5	1	644030	244220	4800	4390	221998.4	404620	626618.6
2	7	4	2	644030	244220	4800	4390	221998.4	244220	466218.4
3	8	4	1	644030	244220	4800	4390	221998.4	643720	686679.6

3	7	5	1	644030	244220	4800	4390	42959.6	404720	447679.6
3	7	4	2	644030	244220	4800	4390	42959.6	244320	287279.6
3	7	5	2	644030	244220	4800	4390	282713.6	4800	287513.6
3	8	4	2	644030	244220	4800	4390	282713.6	243800	526513.6
3	8	5	1	644030	244220	4800	4390	282713.6	404200	686913.6
4	7	5	2	644030	244220	4800	4390	62162.4	4900	67062.4
4	8	4	2	644030	244220	4800	4390	62162.4	243900	306062.4
4	8	5	1	644030	244220	4800	4390	62162.4	404300	466462.4
4	8	5	2	644030	244220	4800	4390	313071.2	4390	317461.2

Table 12: Optimum allocations for 15^{th} stage

m	a	c	i
1	7	4	1
2	7	4	2
3	7	4	2
4	7	5	2

This process continues to obtain the optimum solutions for all the stages that is not possible expressing the calculations of all the stages according to their abundance.

Discussion

The unit costs have a special form in this study because the special structure of the model in the study; Holding cost (C_h) and conduction costs (C_a, C_c, C_i) have less effect on the optimum solutions in comparison with shortage costs (C_{ba}, C_{bc}, C_{bi}). Therefore, shortage costs play the main role in the model strategy and shortage costs for city and industry have more effect in comparison with shortage cost for agriculture. By considering this characteristic, as algorithm continues, optimum solutions for each inventory dam (m) converges to special allocations e.g. for m=4, optimum solution converges to a=7, c=5, i=2. This pattern is not general for all the years definitely and depends rainfall patterns and somewhat demands.

The demands, arrival water and water harvesting have been estimated and forecasted according to regression concept and latest data; if these parameters and variables can be continuous, the solutions will be more precise.

The precipitation of water in this model has been underestimated. But we can exert this factor by raising the accuracy of input parameters and input variables.

Conclusions

This study presented a new method for water allocation by using stochastic dynamic programming. Since this method encompasses the whole of states of allocation and assume the total of affecting parameters in the model, it can be generalized in any region where can face arid condition; but it is sufficient that the geographical conditions and rainfall patterns are considered in the model.

The solutions will be more precise if the continuous distributions can be used for arrival water and demands. Normal distribution can help in these cases. If we want to obtain a more realistic model, we can present different distributions in proportion to each stage for demands; but the calculations will be more extensive. Stochastic dynamic programming can be integrated with metaheuristic methods to obtain more precise and more realistic solutions.

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