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# A Six Degrees of Freedom Robot for Limited Movement of Objects

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**Abstract:** Today, the use of robots and automation practices are the inevitable choices of industries. Considering this ever-increasing demand, the design and analysis of robots as well as the methods used for such purposes are of high importance. This project designs and analyzes a robot with six degrees of freedom used for glass handling purposes. The selected robot has hinge joints with relative less complex kinematic equations. Thanks to its six degrees of freedom, it has wide workspace (work envelope) proportional to its activities. This robot is analyzed using both direct and inverse kinematic methods in order to validate kinematic calculations.

Keywords: direct/inverse kinematic robot, glass handling robot, robotic, workspace

# 1. INTRODUCTION

The emergence of machines applied dramatic changes to human being life so that many works once being done by the physical power of body were assigned to the machines, especially those which are out of the normal abilities of human. The idea of implementing automated systems in factories backs to the World War II. This field has experienced great and considerable changes in recent years so that different robots with different applications can be seen in many research centers, factories and even medical centers. A robot is a mechanical tool used to do different tasks in the industry. This machine can be programmed for different tasks [1,2]. This study evaluates a glass handling robot with six degrees of freedom. Handling glass, especially those with sharp edges, is very risky for human and may seriously threaten the health of workers. Moreover, it is a very sensitive task. Therefore, robots are used in industries to avoid such possible damages and to cover this level of sensitivity. Increased safety of workers, increased productivity and increased energy save are the advantages of the use of a glass handling robot.

# 2- Proposed Robot

Considering the ideal performance of glass handling robot, we tried to select among available robots a robot resembling a mechanical manipulator or gripper with the following specification. This robot is used for training thanks to its ease of use and safety advantages. It was selected because with the mentioned specifications it is one of the most efficient robots manufactured for glass handling with hinge joints which have relatively less complex kinematic equations. In addition, it has wide workspace proportional to glass handling activity thanks to having six degrees of freedom.



Fig. 1 coordinate system attached to a manipulator robot

## **3- Direct Kinematic**

Direct kinematic is actually the calculation of the position and orientation of the end-effector of the manipulator robot. More preciously, given a set of joint angles, direct kinetic is the problem of calculating the position and orientation of the end-effector with respect to the base frame. Sometimes this problem is interpreted as changing the display of the position of a robot from joint space to Cartesian space. In other words, direct kinematic problem establishes a relationship between every joint and the position or status of the end-effector. The proposed glass handling robot has 6 joints 5 of which starting to move at the first step in order to position the end-effector at the considered position. During this process, a fixed position is assumed for the 6<sup>th</sup> joint. The first joint is named as the base joint and the rest of them are named as joint 1, 2, 3, 4, g and 6, respectively.

Coordinate systems are defined as follows using Denavit-Hartenberg method. As one can see, joints are numbered from 1 to n starting from the base joint. In addition, links are numbered from 0 to m so that the base link is numbered  $\theta$ . The so called Inertia  ${}^{0}{}_{0}{}^{x}{}_{0}{}^{y}{}_{0}{}^{z}{}_{0}$  system is fixed on this link. Next, axis  ${}^{z}{}_{0}$  should be selected. It is selected along the rotation axis of the joint. Following the selection of  $z_0$ ,  $z_1$ ,  $z_2$ ,  $z_3$ ,  $z_4$ ,  $z_5$ and  $z_6$  are selected along the rotation axis of their corresponding joints. The latest coordinate system, i.e.  $o_6 x_6 y_6 z_6$ , is assigned to the gripper of the end-effector [3, 4].

parameter	<i>d</i> <sub>1</sub>	<i>a</i> <sub>1</sub>	$a_2$	$a_3$	$d_5$
Corresponding length	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$

**Table 1:** geometrical parameters of the studied robot

$$\begin{split} T_{6}^{0} &= A_{1}A_{2}A_{3}A_{4}A_{5}A_{6} \\ & \begin{bmatrix} c_{\theta_{1}} & 0 & s_{\theta_{1}} & a_{1}c_{\theta_{1}} \\ s_{\theta_{1}} & 0 & -c_{\theta_{1}} & a_{1}s_{\theta_{1}} \\ 0 & 1 & 0 & d_{1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{\theta_{2}} & -s_{\theta_{2}} & 0 & a_{2}c_{\theta_{2}} \\ s_{\theta_{2}} & c_{\theta_{2}} & 0 & a_{2}s_{\theta_{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{\theta_{3}} & -s_{\theta_{3}} & 0 & a_{3}s_{\theta_{3}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \cdot \begin{bmatrix} c_{\theta_{4}} & 0 & s_{\theta_{4}} & 0 \\ s_{\theta_{4}} & 0 & -c_{\theta_{4}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{\theta_{5}} & -s_{\theta_{5}} & 0 & 0 \\ s_{\theta_{5}} & c_{\theta_{5}} & 0 & 0 \\ 0 & 0 & 1 & d_{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{\theta_{6}} & -s_{\theta_{6}} & 0 & 0 \\ s_{\theta_{6}} & c_{\theta_{6}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} s_{1}s_{5} + c_{1}c_{234}c_{5} & s_{1}c_{5} - c_{1}c_{234}s_{5} & c_{1}s_{234} & c_{1}(a_{1} + a_{2}c_{2} + a_{3}c_{23} + s_{234}d_{5}) \\ -c_{1}s_{5} + s_{1}s_{234}c_{5} & -c_{1}c_{5} - s_{1}c_{234}s_{5} & s_{1}s_{234} & s_{1}(a_{1} + a_{2}c_{2} + a_{3}c_{23} + s_{234}d_{5}) \\ s_{234}c_{5} & -s_{234}s_{5} & -c_{234} & d_{1} + a_{2}s_{2} + a_{3}s_{23} - c_{234}d_{5} \end{bmatrix} \end{split}$$

where  $s_{234}$  equals to  $\sin(\theta_2 + \theta_3 + \theta_4)$ . The arrays of matrix  $T_5^0$  are named as follows:

	nx	ox	ax	px
$T^{0} -$	ny	oy	ay	pу
<i>I</i> <sub>6</sub> –	nz,	OZ,	AZ,	pz
	0	0	0	1

#### 4- Inverse kinematic

Apparently, inverse kinematic is slightly more complex than direct one. Given the position and orientation of the end-effector, the process of calculating all possible angles for joints, which can be used to position the robot at the considered position and orientation, is called inverse kinematic.

Direct kinematic deals with determining the position and status of the end-effector in terms of joint variables while inverse kinematic works inversely. In other words, inverse kinematic aims to obtain joint variables versus the final position of the end-effector. This means that given the position of the end-effector, it determines that how the joints should be positioned in order to reach the position of the end-effector [9]. Inverse kinematic equations are obtained as follows using matrix equality.

$$T_{6}^{0} = T_{1}^{0} T_{2}^{1} T_{3}^{2} T_{4}^{3} T_{5}^{4} T_{6}^{5}$$

$$= \begin{bmatrix} s_{1}s_{5} + c_{1}c_{234}c_{5} & s_{1}c_{5} - c_{1}c_{234}s_{5} & c_{1}s_{234} & c_{1}(a_{1} + a_{2}c_{2} + a_{3}c_{23} + s_{234}d_{5}) \\ -c_{1}s_{5} + s_{1}c_{234}c_{5} & -c_{1}c_{5} - s_{1}c_{234}s_{5} & s_{1}s_{234} & s_{1}(a_{1} + a_{2}c_{2} + a_{3}c_{23} + s_{234}d_{5}) \\ s_{234}c_{5} & -s_{234}s_{5} & -c_{234} & d_{1} + a_{2}s_{2} + a_{3}s_{23} - c_{234}d_{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} n_{x} & o_{x} & a_{x} & P_{x} \\ n_{y} & o_{y} & a_{y} & P_{y} \\ n_{z} & o_{z} & a_{z} & P_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5- Jacobian matrix

$$f(x,\theta)=0$$

Mechanisms are constituted by robots attached in series with the end-effector connected to a mobile platform. This set has a series of inputs and outputs which should be determined. The quality of such inputs and outputs depends on the nature of mechanism [5, 6]. In most cases, there is the same number for independent inputs and outputs. This number equals to the degree of freedom of the mechanism. Therefore, the relationship between inputs and outputs is as follows, where f is a D-dimensional implicit function of  $\theta$ , x.

#### 6- Jacobian matrix of the proposed robot

To determine the singular points of the proposed robot, this project needs first to obtain the Jacobian matrix of the robot. To do this, the spatial coordinate vector of the end-effector should be defined and then the partial derivatives of each element should be calculated in terms of the variable of the degrees of freedom of the robot. The elements of x, y and z of the end-effector are as follows:

$$x = c_1(a_1 + a_2c_2 + a_3c_{23} + s_{234}d_5)$$
  

$$y = s_1(a_1 + a_2c_2 + a_3c_{23} + s_{234}d_5)$$
  

$$z = d_1 + a_2s_2 + a_3s_{23} - c_{234}d_5$$

The angles of the end-effector are obtained using Euler equation and the following relations:

$$\theta = a \tan\left(\frac{u_z}{\sqrt{u_x^2 + u_y^2}}\right)$$
$$\varphi = a \tan\left(\frac{u_y}{u_x}\right)$$
$$\psi = a \tan\left(\frac{v_z}{w_x}\right)$$

The parameters of the above relations are replaced using the following matrix.

$$T_6^0 = \begin{bmatrix} u_x & v_x & w_x & x \\ u_y & v_y & w_y & y \\ u_z & v_z & w_z & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Replacing the matrix gives angles of the end-effector as follows:

$$\theta = a \tan\left(\frac{s_{234}c_5}{\sqrt{(s_1s_5 + c_1c_{234}c_5)^2 + (-c_1s_5 + s_1s_{234}c_5)^2}}\right)$$
  
$$\varphi = a \tan\left(\frac{-c_1s_5 + s_1s_{234}c_5}{s_1s_5 + c_1c_{234}c_5}\right)$$
  
$$\psi = a \tan\left(\frac{-s_{234}s_5}{-c_{234}}\right)$$

Therefore, Jacobian matrix is calculated as follows:

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} & \frac{\partial x}{\partial \theta_4} & \frac{\partial x}{\partial \theta_5} & \frac{\partial x}{\partial \theta_6} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} & \frac{\partial y}{\partial \theta_4} & \frac{\partial y}{\partial \theta_5} & \frac{\partial y}{\partial \theta_6} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} & \frac{\partial z}{\partial \theta_4} & \frac{\partial z}{\partial \theta_5} & \frac{\partial z}{\partial \theta_6} \\ \frac{\partial \theta}{\partial \theta_1} & \frac{\partial \theta}{\partial \theta_2} & \frac{\partial \theta}{\partial \theta_3} & \frac{\partial \theta}{\partial \theta_4} & \frac{\partial \theta}{\partial \theta_5} & \frac{\partial \theta}{\partial \theta_6} \\ \frac{\partial \varphi}{\partial \theta_1} & \frac{\partial \varphi}{\partial \theta_2} & \frac{\partial \varphi}{\partial \theta_3} & \frac{\partial \varphi}{\partial \theta_4} & \frac{\partial \varphi}{\partial \theta_5} & \frac{\partial \varphi}{\partial \theta_6} \\ \frac{\partial \varphi}{\partial \theta_1} & \frac{\partial \varphi}{\partial \theta_2} & \frac{\partial \varphi}{\partial \theta_3} & \frac{\partial \varphi}{\partial \theta_4} & \frac{\partial \varphi}{\partial \theta_5} & \frac{\partial \varphi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_3} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_1} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_2} & \frac{\partial \psi}{\partial \theta_4} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_6} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}{\partial \theta_5} \\ \frac{\partial \psi}{\partial \theta_5} & \frac{\partial \psi}$$

The non-zero arrays of this matrix are:

гJ11	J12	J13	J14	0	ן 0
J21	J22	J23	J24	0	0
0	J32	J33	J34	0	0
J41	J42	J43	J44	J45	0
J51	J52	J53	J54	J55	0
Lο	J62	J63	J64	J65	1
	J11 J21 0 J41 J51 0	J11         J12           J21         J22           0         J32           J41         J42           J51         J52           0         J62	J11         J12         J13           J21         J22         J23           0         J32         J33           J41         J42         J43           J51         J52         J53           0         J62         J63	J11         J12         J13         J14           J21         J22         J23         J24           0         J32         J33         J34           J41         J42         J43         J44           J51         J52         J53         J54           0         J62         J63         J64	J11J12J13J140J21J22J23J2400J32J33J340J41J42J43J44J45J51J52J53J54J550J62J63J64J65

#### 7-Workspace

The set of spatial points reachable by the end point is called the workspace of robot. In other words, the determination of the motion range of the output variables for a given range of the inputs is called the workspace. This set of points form a volume which is called the volume of workspace. Considering the number of robot joints and the motion range of each joint, the end-effector reach special points in the space and the set of the points is called the workspace of robot. It is one of the most important specifications of a robot.

Workspace shows the ability of a robot in being positioned in different positions and situations. Therefore, the workspace is of high importance in selecting a robot for industries.

## Different types of workspace with respect to the type of robot are:

Transmission or fixed rotation (fixed direction) workspace, rotating workspace, reachable workspace (maximum space work) and skilled workspace [7]. In direct kinematic, the space work can be calculated by applying all possible angles to every joint of the robot.

# The workspace of glass handling robot

This section first assesses the reachable, or the maximum, workspace and then obtains the skilled workspace of the studied robot. Chapter 2 defines the motion range of each joint of the studied robot. It is assessed using the obtained data. First of all, the overall workspace of the robot is determined.

To display the overall workspace of the robot, the angle of each joint is varied from the minimum to the maximum possible value. Fig. 2 uses this method and shows the overall workspace of the robot in the form of scattered points.



Fig. 2 workspace of robot

When angles  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  shift from their minimum value to the maximum value, the reachable space of the robot will be as per Fig. 3. This figure shows the decreased reach of the robot to its total workspace.



Fig. 3: workspace of robot versus the first three input angles

Similarly, by varying  $\theta_1$ ,  $\theta_2$  and  $\theta_4$ , the workspace of the robot will be as per Fig. 4. If  $\theta_1$ ,  $\theta_2$  and  $\theta_4$  shift from their minimum value to their maximum value and other angles is remained fixed, the reachable space will be as follows:



**Fig. 4:** workspace of robot versus  $\theta_1$ ,  $\theta_2$  and  $\theta_4$ For the variations of  $\theta_2$ ,  $\theta_3$  and  $\theta_4$ , the reachable space will be as per Fig. 5.



Fig. 5: workspace of robot versus  $\theta_1$  ,  $\theta_2$  and  $\theta_4$ 

When  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  are at their minimum value and only the base angle rises from its minimum value (-160) to the maximum value (+160), the reachable space of the robot will be as follows.  $\theta_5$  and  $\theta_6$  are the roll angle of the end-effector and the gripper. Therefore, their value does not affect the position of the end-executor in the work space.



Fig. 6 workspace of the end-effector versus the rotation of joint 1

If  $\theta_1$ ,  $\theta_3$  and  $\theta_4$  are fixed at their minimum value and  $\theta_2$  shifts from its minimum value (-30) to the maximum value (+120), the workspace of the end-effector will be changed to Fig. 6.



Fig. 7: variation of the position of the end-effector versus the rotation of the shoulder joint

To display the motion range of joint 3,  $\theta_1$ ,  $\theta_2$  and  $\theta_4$  are set at their minimum value and the third angle is changed from the minimum value (-130) to the maximum value (+130) using direct dynamic code. In this way, the workspace will be as per Fig. 7.



Fig. 8: variation of the position of the end-effector versus the rotation of the elbow joint



Fig. 9: the workspace of the end-effector versus the rotation of joint 4

The same process is repeated in joint 4 where  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  are set at their minimum value and  $\theta_4$  shifts from its minimum value (-130) to the maximum value (+130). This results in the workspace shown in Fig. 8. As it was mentioned before, changes in the roll angle of joint 5 do not change the position of the end-effector. Therefore, the changes of the angles of joint 5 were not studied here.

In fig. 10, only one joint moves in every turn as follows. At firs  $\theta_4$  shifts from+120 to -120. Then,  $\theta_3$  repeats the same range,  $\theta_2$  shifts from -30 to +120 and finally  $\theta_1$  shifts from -160 to +160.



Fig. 10 the trajectory of the end-effector of the studied robot

## 8- Conclusion

This study first modeled the problem-solving of the workspace of a robot with six degrees of freedom installed in auto-making factory used for handling glass. Then, the robot was designed considering the conditions to be

taken into account for glass handling. In the process of modeling, the kinematic model was calculated considering the study assumptions in order to take into considerations the glass safety as well as the effects of glass weight or stimulus induced by robot maneuver to glass handling. The values associated with joint parameters and movements and ideal trajectories for kinematic implementation of the robot were presented. Then, different types of workspace and methods for obtaining them were defined using Jacobian matrix. A workspace sample was presented and relevant curves of robot motion and workspace range were drawn versus each angle. In addition, the spatial points reachable by the end-effector of the robot were studied and it was indicated that all possible angles of the robot joints as well as the workspace of robot can be calculated using direct dynamic. Then, the simulation equations were extracted for glass handling purposes. This was analyzed by direct and inverse analyses separately-robot analysis requires both. Given joint variables, the position and status of the end-effector were calculated using direct dynamic and kinematic. It should be noted that joint variables are determined based on the positions of the glass. Based on joint variables i.e. the angles between links and the increase of length, direct dynamic and direct kinematic problems were studied and the obtained results were used to validate the problem. Indirect dynamic and kinematic were the other side of the problem which were used following relevant analyses. The inverse dynamic and kinematic, which are used to accurately calculate that position of joints which is the ideal position of the end-effector, were used to validate the direct problem. As it was seen, considering the results of dynamic and kinematic problems, the studied robot with six degrees of freedom can well work in a proper workspace. Considering the outcomes of this project, the following studies are suggested for future research:

- Adopting a fit controlled for language control
- Applying the effect of uncertainties on the system
- Adopting modern controllers to obtain better results in nullifying errors
- Moving glass considering the obstacles in the way of robots sophisticating robot trajectory
- Adopting two glass handling robots at the same time
- Designing glass cleaner robots

The optimum trajectory for handling glass from storage to machine feed was selected based on inverse kinematic equations.

# REFERENCES

- 1. Okada, K. And Kojima M. "Vision based behavior verification system of humanoid robot for daily environment tasks", International World 6<sup>th</sup> Conf. on Humanoid Robots IEEE-RAS, pp. 7-12, 2006.
- 2. Cutkosky, A. and Mark, R. "On grasp choice, grasp models, and the design of hands for manufacturing tasks", Robotics and Automation, IEEE Transactions on 5, 269-279, 1989.
- 3. Salisbury, A.B. and Dario P. "Augmentation of grasp robustness using intrinsic tactile sensing", International Conf. on Robotics and Automation, pp. 302-307, 1989.
- 4. Jacobsen, S.C. John, E. Wood, Knutti, D.F. and Biggers, K.B. "The UTAH/MIT dextrous hand: Work in progress", The International Journal of Robotics Research, vol. 3 pp. 21-50, 1984.
- 5. Luo, R.C. and Fukuda, T. "Special issue on networked intelligent robots through the internet [Scanning the Issue]", Proceedings of the IEEE 91, pp. 367-370, 2003.
- 6. Kwan, S.S. Park, J. B. and Choi, Y. H. "Dual-Fingered stable grasping control for an optimal force angle", Transactions on Robotics, IEEE, pp. 256-26, 2012.
- 7. Ananthraman, S. and Garg, D.P. "Training backpropagation and CMAC neural networks for control of a SCARA robot", Engineering Applications of Artificial Intelligence vol.6, pp.105-115, 1993.
- 8. Tomohiro, T. and Sugeno, M. "Fuzzy identification of systems and its applications to modeling and control", Systems, Man and Cybernetics, IEEE Transactions on,vol. 1, pp.116-132, 1985.
- 9. Do, W.Q.D. and D. Yang C.H. "Inverse dynamic analysis and simulation of a platform type of robot", Journal of Robotic Systems, vol. 5, pp. 209-227, 1988.