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Folding and unfolding of chaotic spheres in chaotic space-like Minkowski space-Time

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ABSTRACT: In this paper we introduce types of foldings and unfoldings of chaotic spheres in chaotic space-like Minkowski space- time into itself. The classification of these types of foldings and unfoldings of chaotic spheres are deduced. The relations between the foldings and unfoldings of chaotic spheres are presented. The limits of foldings and unfoldings of chaotic spheres are discussed. Types of retractions of chaotic spheres in chaotic space-like Minkowski space- time are also obtained.
Keywords: Minkowski space-Time; Chaotic spheres; Folding; Retraction.
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Introduction And Definitions

Chaos theory is the branch of mathematics for the study of processes that seem so complex that at first they do not appear to be governed by any known laws or principles, but which actually have an underlying order that can be described by vector calculus and its associated geometry. Examples of chaotic processes include a stream of rising smoke that breaks down and becomes turbulent, water flowing in a stream or crashing at the bottom of a waterfall, electroencephalographic activity of the brain, changes in animal populations, fluctuations on the stock exchange, and the weather (either local or global). All of these phenomena involve the interaction of several elements and the pattern of their changes over time(El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c).

An n-dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n.If $h:U \rightarrow U'$ is a homeomorphism of $U \subseteq M$ onto $U' \subseteq R^n$,then h is called a chart of M and U is the associated chart domain. A collection (h_{α,U_α}) is said to be an atlas for M if $\cup_{\alpha\in A} U_\alpha = M$. Given two charts h_α, h_β such that $U_{\alpha\beta}=U_\alpha \cap U_\beta \neq \emptyset$,the transformation chart $h_\beta \circ h_\alpha^{-1}$ between open sets of R^n is defined ,and if all of these charts transformation are c^∞-mappings, then the manifolds under consideration is a c^∞ - manifolds .A differentiable structure on M is a differentiable atlas and a differentiable manifolds is a topological manifolds with a differentiable structure (Arkowitz M. 2011), (Reid M, Balazs S. 2005), (Shick Pl. 2007) and (Strom J. 2011). M may have another structures as colour ,density or any physical structures. The number of structures may be infinite. In this case the manifold is said to be a chaotic manifold (El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c). and may become relevant to vacuum fluctuation and chaotic quantum field theories (El-Ahmady A .E. 2007a).The magnetic field of a magnet bar is a kind of chaotic 1-dimensional manifold represented by the magnetic flux lines . The geometric manifold is the magnetic bar itself. Fuzzy manifolds are special type of the category of chaotic manifolds(El-Ahmady A .E. 2007b), (El-Ahmady A .E. 2004a), (El-Ahmady A .E. 2004b)and(El-Ahmady A .E. 2006) .Usually we denote by $M=M_{012\ldots h}$ to a chaotic manifolds(El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c), where M_{0h} is the geometric(essential) manifold and the associated pure chaotic manifolds, the manifolds with physical characters, are denoted by M_{1h} ,…, $M_{\infty h}$.

Most folding problems are attractive from a pure mathematical standpoint, for the beauty of the problems themselves(El-Ahmady A. E. 2011a), (El-Ahmady A. E. 2011b). The folding problems have close connections to important industrial applications. Linkage folding has applications in robotics and hydraulic tube bending. Paper folding has application in sheet-metal bending, packaging, and air –bag folding(El-Ahmady A. E. 2013a)and(El-Ahmady A. E. 2012).Also used folding to solve difficult problems related to shell structures in civil engineering and aero space design, namely buckling instability(El-Ahmady A. E. 2013c).Isometric folding between two Riemannian manifold may be characterized as maps that send piecewise geodesic segments to a piecewise geodesic segments of the same length (El-Ahmady A. E. 2013d).For a topological folding the maps do not preserves lengths(El-Ahmady A. E. 1949)and (El-Ahmady A. E. 2011b) .

Let M and N be two Riemannian manifolds of the same dimensions, a map $g: M \longrightarrow N$ is said to be unfolding of M into N, if, for every piecewise geodesic path $\gamma: I \longrightarrow M$ (I=[0,1]⊆ R), the induced path $\gamma' = g \circ \gamma: I \longrightarrow N$ is a piecewise geodesic but with length greater than that of γ, i.e.,
$$\forall\, x, y \in M \Rightarrow d(x,y) \le d\big(g(x), g(y)\big)$$ (El-Ahmady A. E. 2011b) .

A subset A of a topological space X is called a retract of X if there exists a continuous map $r: X \longrightarrow A$ such that $r(a) = a \ \forall\, a \in A$, where A is closed and X is open (Naber GL. 2011),

Let $\overline{M}^{n+1} \cong M_{012\ldots h}$ be the Minkowski (n+1)-space ,that is, $\overline{M}^{n+1}$ is the real vector space $\tilde{R}^{n+1}$ endowed with the standard flat metric $ds^{\tilde{\tilde{2}}} = \sum_{i=1}^{n-1} dx_i^{\tilde{\tilde{2}}} - dx_n^{\tilde{\tilde{2}}}$. Also , $ds^{\tilde{\tilde{2}}} > 0$, $ds^{\tilde{\tilde{2}}} = 0$ and $ds^{\tilde{\tilde{2}}} < 0$ correspond to space –like ,null and time –like geodesics (James B, Hartle G. 2003).In this paper we will introduce a new type of foldings of chaotic spheres in chaotic space-like Minkowski space time from view point of the variation of the density on chaotic spheres in chaotic space-like Minkowski space time and retraction of chaotic black hole, as presented by (El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c).

The Main Results

The chaotic Minkowski space $\overline{M}^{n+1} \cong M_{012\ldots h}$has three types of chaotic vector fields, chaotic space-time, chaotic null cone and chaotic space-like. In the chaotic space-like $\overline{M}^{n+1}$ the chaotic spheres $S^n_{r_{1\infty h}}$ take the form

$$\{x/(x_{10},0)^2 + (x_{20},0)^2 + \cdots + (x_{n0},0)^2 - (x_{(n+1)0},0)^2 = (r_{10})^2\},$$

$$(x_{10},\mu_1)^2 + (x_{20},\mu_1)^2 + \cdots + (x_{n0},\mu_1)^2 - (x_{(n+1)0},\mu_1)^2 = (r_{11h})^2,$$

$$(x_{10},\mu_1)^2 + (x_{20},\mu_1)^2 + \cdots + (x_{n0},\mu_1)^2 - (x_{(n+1)0},\mu_1)^2 = (r_{12h})^2, \cdots,$$

$$(x_{10},\mu_1)^2 + (x_{20},\mu_1)^2 + \cdots + (x_{n0},\mu_1)^2 - (x_{(n+1)0},\mu_1)^2 = (r_{1kh})^2, \cdots,$$

$$(x_{10},\mu_1)^2 + (x_{20},\mu_1)^2 + \cdots + (x_{n0},\mu_1)^2 - (x_{(n+1)0},\mu_1)^2 = (r_{1\infty h})^2.$$

Now, we will discuss the folding of the geometric sphere $S^n_{r_{10}}$ into itself. Let $f: S^n_{r_{10}} \longrightarrow S^n_{r_{10}}$ be defined by

$$f_1\{(x_{10},0)^2 + (x_{20},0)^2 + \cdots + (x_{n0},0)^2 - (x_{(n+1)0},0)^2 = (r_{10})^2\}$$
$$= \{|(x_{10},0)|^2 + |(x_{20},0)|^2 + \cdots + |(x_{n0},0)|^2 - |(x_{(n+1)0},0)|^2 = (r_{10})^2\},$$

$$f_2\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\}$$
$$=\{\frac{|(x_{10},0)|^2}{2}+\frac{|(x_{20},0)|^2}{2}+\cdots+\frac{|(x_{n0},0)|^2}{2}-\frac{|(x_{(n+1)0},0)|^2}{2}=(r_{10})^2\},\cdots,$$
$$f_N\{\frac{|(x_{10},0)|^2}{N-1}+\frac{|(x_{20},0)|^2}{N-1}+\cdots+\frac{|(x_{n0},0)|^2}{N-1}-\frac{|(x_{(n+1)0},0)|^2}{N-1}=(r_{10})^2\}$$
$$=\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\}.$$

Now, we are going to discuss the following cases:

(1) If $1<N<\infty$, in this case f_N represents a folding of the geometric sphere $S^n_{r_{10}}$ into itself. This folding induces the geometric hypersphere $\bar{S}^n_{r_{10}}$ in Minkowski-space like M^{n+1}. Also, there are new types of limits of the foldings of the geometric sphere $S^n_{r_{10}}$ given by

$$\lim_{N\to\infty} f_N\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\}$$
$$=(\varepsilon_1^2,\varepsilon_2^2,\ldots,0,\varepsilon_{j+1}^2,\varepsilon_{j+2}^2,\ldots,\varepsilon_{(n+1)}^2),$$

where $\varepsilon_j\to 0$ is faster than any one of $\varepsilon_i, i\neq j, i,j=1,2,\cdots,n$.

Also, $$\lim_{N\to\infty} f_N\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\}$$
$$=(\varepsilon_1^2,\varepsilon_2^2,\ldots,0,\varepsilon_{j+1}^2,\varepsilon_{j+2}^2,0,\ldots,\varepsilon_{(n+1)}^2),$$

where more than one ε tends to zero, $\varepsilon_j,\varepsilon_{j+3}\to 0$.Moreover,

$$\lim_{N\to\infty} f_N\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\}$$
$=(0,0,\cdots,0,0,0,\cdots,0)$, it is the origin or coinsides with the null cone. Also,

$$\lim_{N\to\infty} f_N\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\}\}$$
$=(\varepsilon_1,\varepsilon_2,\delta,\varepsilon_4,\cdots,\ \varepsilon_{n+1})$, where $\delta<<<1$ and $\delta<<\varepsilon_i, \forall_i=1,2,\cdots,(n+1)$.

The limit of foldings in this case represents the fractal dimension of the hypersphere $\bar{S}^{(n-1)+\frac{1}{m}}_{r_{10}}=\bar{S}^{n-\frac{1}{p}}, m>>>1$ in Minkowski space-like M^{n+1} which is called fractal folding of the hypersphere $\bar{S}^{n}_{r_{10}}$ into $\bar{S}^{n-\frac{1}{p}}_{r_{10}}$. Moreover,

$$\lim_{\delta\to 0}\left\{\lim_{N\to\infty} f_N\left\{\frac{|(x_{10},0)|^2}{N}+\frac{|(x_{20},0)|^2}{N}+\cdots+\frac{|(x_{n0},0)|^2}{N}-\frac{|(x_{(n+1)0},0)|^2}{N}=(r_{10})^2\right\}\right\}$$

$=\lim_{\delta\to 0}\{(\varepsilon_1,\varepsilon_2,\delta,\varepsilon_4,\cdots,\ \varepsilon_{n+1})=(\varepsilon_1,\varepsilon_2,0,\varepsilon_4,\cdots,\ \varepsilon_{n+1})$, which is the hypersphere $\bar{S}^{n-1}\subset M^n$ which is the limit of fractal foldings.

Thus the above result can be formulated in the following theorem.

Theorem 1. The folding of any manifold H^n into itself preserves the dimension n, while the folding of hypersphere in Minkowski space-like M^n into itself does not preserve the dimension of Minkowski space-like M^n. It will be a fractal dimension $n\pm\frac{1}{p}$.

(ii) If $0<N<1$, in case (i) f_N is a type of folding but in this case f_N is a type of unfolding, i.e., f_N is an equivalent to un f_N,

$$\text{un } f_1\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\}$$
$$=\frac{1}{2}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\},$$
$$\text{un } f_2\{\frac{1}{2}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2\}=(r_{10})^2\},$$
$$=\frac{1}{3}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\},\cdots,$$
$$\text{un } f_{N-1}\{\frac{1}{N-1}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\}\}$$
$$=\frac{1}{N}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\}.$$

Hence, we approach the limit of the unfolding of the geometric sphere $S^{n}_{r_{10}}$ in Minkowski space-like M^{n+1},

$$\lim_{N\to 0}\text{un } f_N\{\frac{1}{N}\{|(x_{10},0)|^2+|(x_{20},0)|^2+\cdots+|(x_{n0},0)|^2-|(x_{(n+1)0},0)|^2=(r_{10})^2\}\}$$
$$=\begin{cases}(\eta_1,\eta_2,\cdots,\eta_{n+1}) & or\\ (y_1^2,y_1^2,\cdots,y_n^2,y_{n+1}^2) & or\\ (y_1^2,y_1^2,\cdots,y_n^2,y_{n+1}^2,\varepsilon_1) & or\\ (y_1^2,y_1^2,\cdots,y_n^2,y_{n+1}^2,\varepsilon_n),\ \varepsilon_n>\varepsilon_{n-1}<\varepsilon_{n-2}>\cdots>\varepsilon_1.\end{cases}$$

Also, $\lim\limits_{\varepsilon_n \to \infty}\{\lim\limits_{N \to 0} \text{un}\, f_N \{\frac{1}{N}\{|(x_{10},0)|^2 + |(x_{20},0)|^2 + \cdots + |(x_{n0},0)|^2$

$$-|(x_{(n+1)0},0)|^2 = (r_{10})^2 \}\}\} = \lim_{\varepsilon_n \to \infty}(y_1^2, y_1^2, \cdots, y_{n+1}^2, \varepsilon_n)$$

$$= (y_1^2, y_1^2, \cdots, y_n^2, y_{n+1}^2, y_{n+2}^2),$$

It is the limit of fractal unfoldings of the sphere which will be $\bar{S}^{n+1}_{r_{10}}$ in Minkowski space-like M^{n+2}.

Thus, the following theorem is obtained

Theorem 2. The unfolding of any manifold H^n into itself preserves the dimension n while in hyperspheres in Minkowski space-like M^n the unfolding will increase the dimension till it be a fractal $M^{n+\varepsilon}$. In the limit case it will be an integer $n+1$.

Corollary 1. The end of the limit of unfoldings will be all the space-like which is represented by a maximum sphere $S^{n+1} \subset M^{n+2}$.

Consider the chaotic spheres $S^n_{r_{01\infty h}}$ in Minkowski space-like M^{n+1}, in symbols $S^n_{r_{01\infty h}} \equiv S^n_{01234\cdots\infty h}$, which is covered by the physical character $\mu_1, \mu_2, \mu_3, \mu_4, \cdots, \mu_\infty$, then the following cases of the foldings of the physical characters into themselves are given by

(1) The folding $f_{h_i}: S^n_{01234\cdots\infty h} \longrightarrow S^n_{01234\cdots\infty h}$, $i = 1, 2, 3, \cdots, \infty$, such that

$$f_{h_1}: S^n_{01234\cdots\infty h} \longrightarrow S^n_{0234\cdots\infty h},$$

$$f_{h_2}: S^n_{01234\cdots\infty h} \longrightarrow S^n_{0134\cdots\infty h},$$

$$f_{h_3}: S^n_{01234\cdots\infty h} \longrightarrow S^n_{0124\cdots\infty h},\cdots,$$

$$f_{h_i}: S^n_{01234\cdots\infty h} \longrightarrow S^n_{01234\cdots,i+1,\cdots\infty h}.$$

This means that the i physical character is folded in the $(i+1)$ physical character, see Figure (1).

Figure (1)

(2) The folding $\bar{f}_{h_i}: S^n_{01234\cdots\infty h} \longrightarrow \bar{S}^n_{01234\cdots\infty h}$, such that

$$\bar{f}_{h_1}: S^n_{01234\cdots\infty h} \longrightarrow \bar{S}^n_{0\bar{1}234\cdots\infty h},$$

$$\bar{f}_{h_2} : S^n_{01234\cdots\infty h} \longrightarrow \bar{S}^n_{01\bar{2}34\cdots\infty h},$$

$$\bar{f}_{h_3} : S^n_{01234\cdots\infty h} \longrightarrow \bar{S}^n_{012\bar{3}4\cdots\infty h}, \ldots,$$

$$\bar{f}_{h_i} : S^n_{01234\cdots\infty h} \longrightarrow \bar{S}^n_{01234\cdots,\overline{i+1},\cdots\infty h}.$$

This means that the i physical character of the chaotic spheres $S^n_{01234\cdots\infty h}$ folded into the $\overline{(i+1)}$ physical character of the upper chaotic spheres $\bar{S}^n_{01234\cdots,\overline{i+1},\cdots\infty h}$, see Fig. (2-a).

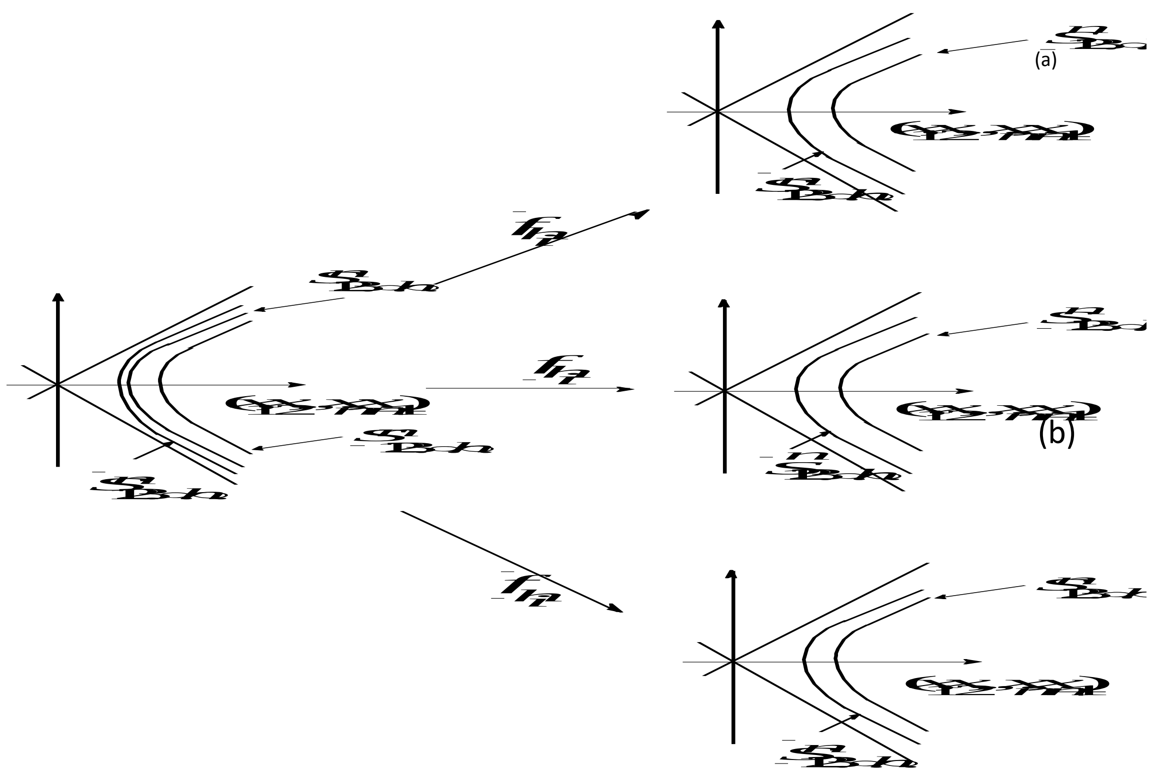

Figure (2)

(3) The folding $\underline{f}_{h_i} : S^n_{01234\cdots\infty h} \longrightarrow \underline{S}^n_{01234\cdots\infty h}$, $i = 1, 2, 3, \cdots, \infty$, such that

$$\underline{f}_{h_1} : S^n_{01234\cdots\infty h} \longrightarrow \underline{S}^n_{0\bar{1}234\cdots\infty h},$$

$$\underline{f}_{h_2} : S^n_{01234\cdots\infty h} \longrightarrow \underline{S}^n_{01\bar{2}34\cdots\infty h},$$

$$\underline{f}_{h_3} : S^n_{01234\cdots\infty h} \longrightarrow \underline{S}^n_{012\bar{3}4\cdots\infty h}, \ldots,$$

$$\underline{f}_{h_i} : S^n_{01234\cdots\infty h} \longrightarrow \underline{S}^n_{01234\cdots,\overline{i+1},\cdots\infty h}.$$

This folding is the i physical character of the chaotic spheres $S^n_{01234\cdots\infty h}$ into the $\overline{(i+1)}$ physical character of the lower chaotic spheres $\underline{S}^n_{01234\cdots,\overline{i+1},\cdots\infty h}$, see Fig. (2-b).

(4) The folding of all physical characters of chaotic spheres $\bar{S}^n_{01234\cdots\infty h}$ are folded into the chaotic spheres $\underline{S}^n_{01234\cdots\infty h}$, i.e., $\underline{\bar{f}}_{h_i} : S^n_{01234\cdots\infty h} \longrightarrow S^n_{01234\cdots,\overline{i+1},\cdots\infty h}$ such that $\underline{\bar{f}}_{h_i}(\bar{S}^n_{01234\cdots\bar{i}\cdots\infty h}) = \underline{\bar{S}}^n_{01234\cdots\bar{i}\cdots\infty h}$, See Fig. (2-c).

From the above mentioned approach the following result is presented.

Theorem 3. The chaotic foldings of the system of the chaotic spheres in chaotic Minkowski space-like M^{n+1} into itself is divided into three cases:

(i) Folding of the lower system into itself.
(ii) Folding of the upper system into itself.
(iii) Folding of the upper system into lower system or vice-versa.

Consider the geometric sphere $S^n_{r_{10}}$ in Minkowski space-like M^{n+1} with geometric retractions

$$rg_1 : S^n_{r_{10}} \longrightarrow S^{n-1}_{r_{10}}, \quad rg_2 : S^{n-1}_{r_{10}} \longrightarrow S^{n-2}_{r_{10}}, \cdots,$$

$$rg_S : S^2_{r_{10}} \longrightarrow S^1_{r_{10}}, \quad rg_{S+1} : S^1_{r_{10}} \longrightarrow S^0_{r_{10}},$$ where $S^0_{r_{10}}$ is the origin or any point on the null cone and $S^n_{r_{10}}, S^{n-1}_{r_{10}}, \cdots, S^2_{r_{10}}, S^1_{r_{10}}$ are open spheres in Minkowski space-like M^{n+1} or in n-dimensional Euclidean space R^n. Now, consider the chaotic sphere $(S^n_{r_{kh}}, \mu)$ in Minkowski space-like M^{n+1} with chaotic retractions

$$r_h : (S^n_{r_{kh}}, \mu_1) \longrightarrow (S^n_{r_{kh}}, \mu_2), r_h(S^n_{r_{kh}}) = S^n_{r_{kh}}, r_h(\mu_1) = (\mu_2), \quad \mu_2 \subset \mu_1,$$

this retraction restricted on the density of the physical character but the geometric sphere are the same. Also, let

$$r_{h_1} : (S^n_{r_{kh_1}}, \mu_1) \longrightarrow (S^{n-1}_{r_{kh_2}}, \mu_2),$$

$$r_{h_2} : (S^{n-1}_{r_{kh_2}}, \mu_2) \longrightarrow (S^{n-2}_{r_{kh_3}}, \mu_3), \ldots,$$

$$r_{h_s} : (S^2_{r_{kh_s}}, \mu_s) \longrightarrow (S^1_{r_{kh_{s+1}}}, \mu_{s+1}),$$

$$r_{h_{s+1}} : (S^1_{r_{kh_{s+1}}}, \mu_{s+1}) \longrightarrow (S^0_{r_{kh_{s+2}}}, \mu_{s+2}), \quad s = 1, 2, \cdots, n, \cdots \infty,$$

$\mu_{s+2} \subset \mu_{s+1} \subset \mu_s \subset \cdots \subset \mu_2 \subset \mu_1$, then $\lim_{s\to\infty} r_{h_s}(S^2_{r_{kh_s}}, \mu_s) = (S^1_{r_{kh_\infty}}, \mu_\infty)$.

which is the chaotic open spheres in chaotic Minkowski space-like M^2 or chaotic Euclidean space R^2. Also, $\lim_{s\to\infty} r_{h_s}(S^1_{r_{kh_{s+1}}}, \mu_{s+1}) = (S^0_{r_{kh\infty}}, \mu_\infty)$, the retraction of $(S^0_{r_{kh\infty}}, \mu_\infty)$ is not defined which is the chaotic closed spheres.
This retraction deals with the imagation space, then the limit of the chaotic retractions does not induce the geometric retraction. Also,
$\lim_{s\to\infty} r_{g_{s+1}}(S^1_{r_{10}}, \mu_{s+1}) = (S^0_{r_{10}}, \mu_\infty)$. It is the end of the retractions and $r_{g_{s+2}}(S^0_{r_{kh\infty}}, \mu_\infty)$ is not defined.
Hence, the limit of the geometric retraction does not affect the density of the physical character.
From the above approach the following results are obtained.

Corollary 2. The retraction $r : X \longrightarrow A$, A must be closed and X must be open. In this case X is open and A is open we will call it open retraction.

Theorem 4. The limit of the retraction of any geometric spheres in Minkowski space-like is $(S^0_{r_{10}}, \mu_s)$ which is closed. But limit of density function μ_s does not deduce the geometric limit.

Consider the chaotic open spheres $(S^n_{r_{kh}}, \mu_s), \mu_s$ is the density of the physical character. Now, we will discuss the following types of retraction

(i) The first type of chaotic retraction is defined by

$$r_{h_{11}} : (S^n_{r_{kh}}, \mu_1) \longrightarrow (S^n_{r_{kh}}, \mu_2),$$

$$r_{h_{12}} : (S^n_{r_{kh}}, \mu_2) \longrightarrow (S^n_{r_{kh}}, \mu_3), \ \ldots,$$

$$r_{h_{1s}} : (S^n_{r_{kh}}, \mu_s) \longrightarrow (S^n_{r_{kh}}, \mu_{s+1}),$$

$$\mu_{s+1} < \mu_s < \mu_{s-1} < \mu_{s-2} < \cdots < \mu_1 < \mu.$$

This type of chaotic retractions is a retraction of the density of the physical character at every point. Also, these retractions represent a type of foldings of the chaotic spheres, see Fig. (3)

Figure (3)

Also, $\lim_{s\to\infty} r_{h_{1s}}(S^n_{r_{kh}}, \mu_s)$ does not lead to geometric retraction.

(ii) The second type of the chaotic retractions of the density of the physical character is

$r_{h_{21}} : (S^n_{r_{kh}}, \mu_1) \longrightarrow (S^n_{r_{kh}}, \mu_2)$,

$r_{h_{22}} : (S^n_{r_{kh}}, \mu_2) \longrightarrow (S^n_{r_{kh}}, \mu_3)$, ...,

$r_{h_{2s}} : (S^n_{r_{kh}}, \mu_s) \longrightarrow (S^n_{r_{kh}}, \mu_{s+1})$,

$\mu_{s+1} \subset \mu_s \subset \cdots \subset \mu_2 \subset \mu_1 \subset \mu$.

This means that the density will be collected in the right half of the spheres to piecewise segements. See Fig. (4) and $\lim_{s\to\infty} r_{h_{2s}}(S^n_{r_{kh}}, \mu_s) = \mu_d$. Moreover, these retractions represent a type of chaotic folding of chaotic spheres.

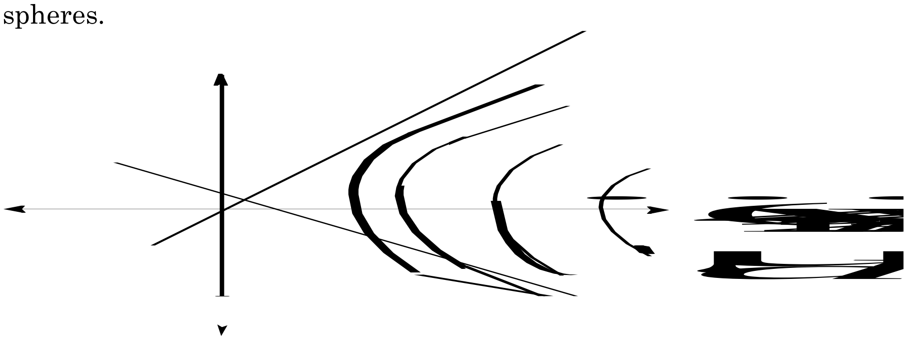

Figure (4)

(iii) This type of retractions is the geometric retractions of the open spheres $(S^n_{r_{kh}}, \mu_s)$ defined by

$r_{g_{h_{31}}} : (S^n_{r_{kh}}, \mu_s) \longrightarrow (S^{n-1}_{r_{kh}}, \mu_s)$,

$r_{g_{h_{32}}} : (S^{n-1}_{r_{kh}}, \mu_s) \longrightarrow (S^{n-2}_{r_{kh}}, \mu_s)$, ...,

$r_{g_{h_{3s}}} : (S^2_{r_{kh}}, \mu_s) \longrightarrow (S^1_{r_{kh}}, \mu_s)$.

Also, the limit of sequence of geometric retractions will be a point of the same density μ_s, $\lim_{s\to\infty} r_{g_{h_{3s}}}(S^2_{r_{kh}}, \mu_s) = (S^1_{r_{kh}}, \mu_s)$. See Fig. (5). Also, in this case the retractions do not induce a type of foldings.

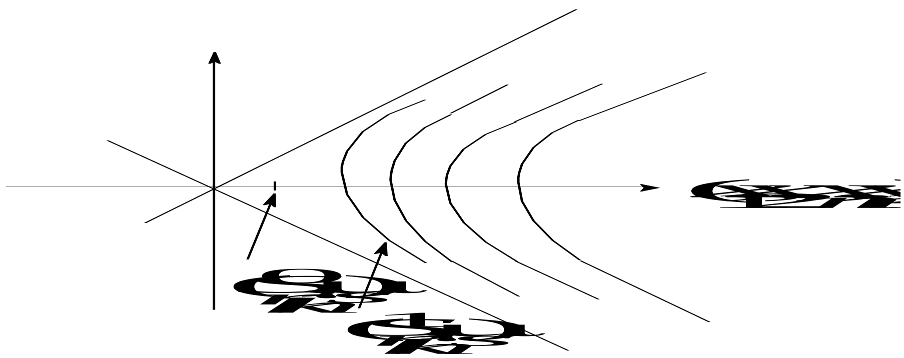

Figure (5)
Hence, we are in a position to formulate the following theorem;
Theorem 5. The first and second types of retractions are types of foldings but the third one is not a type of foldings.

Applications

(i) There are many applications to the density functions. In the field of medicine, for instance, we find that the patient who suffers from cancer which leads to many other sub-diseases, in the heart, kidney, blood, prostat, skin and nearly in all organs. That, who is attacked by the hepatitic virus C , in his body suffers from many symptoms as a result of his liver troubles. Also, in the field of medicine, the duzzeling of the eyes…. The eye can see fish in the air, and some real figures in the air if the person concerned is un will, the phenomen represented by some one of the matrices g_1, g_0, g_2 where g_1 inside the cone, g_0 on the come, g_2 is outside the cone.

Mathematically, these factors and symptoms are expressed by the density functions where $(C(\lambda_1, \lambda_2, \ldots, \lambda_n), \mu_1, \mu_2, \ldots, \mu_n), \mu_1 \in [0,1], \ \mu_2, \mu_3, \ldots, \mu_n \in [-1,1]$ the effective functions(El-Ahmady A .E. 2007a).

(ii) The geometric metric function represents real phenomen if the metric is a Riemannian metric, but in the case of a pseudo-Riemannian metric it will be applied in phenomena which is not real like the complex potential function and complex radiation(El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c).

(iii) The Ritz variational method during the calculation of the ground – state energy in a fuzzy framework. Consider a Hamilton H, and an arbitrary square integrable function Ψ , so that $\langle \Psi / \Psi \rangle = 1$. Considering Ψ as a fuzzy function and the ranking system as defined in (El-Ahmady A. E. 2011c),similar to (El-Ahmady A. E. 2011c),it can be shown that $\langle \psi / H / \psi \rangle$ is a fuzzy upper bound on E_0 (ground-stat energy). Now $\langle \psi / H / \psi \rangle$ should be minimizing the distance between E_0 and respect to a number of parameters $(\alpha_1, \alpha_2, \ldots)$. This can be done by minimizing distance between E_0 and $\langle \psi / H / \psi \rangle$. The rest of the discussion is the same as that provided in (El-Ahmady A .E. 2007a)and (El-Ahmady A. E. 2011c).

Conclusions

In this paper we have presented the foldings and unfoldings of the chaotic spheres in chaotic space-like Minkowski space time. The variation of dimensions under the folding and unfolding of chaotic spheres in chaotic space-like are deduced. The classification of the chaotic foldings of the system of the chaotic spheres in chaotic Minkowski space are discussed. Also, the connection between the retractions and foldings of the chaotic spheres are presented. Some applications are discussed.

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