Science Arena Publications
Specialty Journal of Architecture and Construction
Available online at www.sciarena.com
2015, Vol, 1 (1): 14-20

The deformation retract of time like helix in Minkowski 3-space

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ABSTRACT: Our aim in the present article is to introduce and study new types of retraction of time like helix in Minkowski 3-space.Types of the deformation retracts of the time like helix in Minkowski 3-space are presented. The relations between the retraction and the deformation retract of time like helix are deduced. Types of minimal retraction of time like helix in Minkowski 3-space are also presented . New types of homotopy maps are described.
Keywords: Minkowski 3-space, time like helix, retractions, deformation retracts.
Mathematics Subject Classification: 53A35, 51H05, 58C05, 51F10.

Introduction

As is well known, the theory of deformation retracts is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry[1-21] .

Helix is one of the most fascinating curves in Science and Nature .From the view of differential geometry ,a helix is a geometry curve with non –vanishing constant curvature (or first curvature of the curve and denoted by k_1) and non-vanishing constant torsion (or second curvature of the curve and denoted by k_2). Indeed a helix is a special case of the general helix .A curve of constant slope or general helix in Euclidean 3-space E^3 ,is defined by the property that the tangent makes a constant angle with a fixed straight line [14-17]. It is well known that to each unit speed curve $\alpha : I \subset R \to R^3$ in the Euclidean space R^3 whose successive derivatives $\alpha'(s)$, $\alpha''(s)$ and $\alpha'''(s)$are linearly independent vectors ,one can associate three mutually orthogonal unit vector field T,N and B called respectively the tangent ,the principle normal and the binormal vector field [22] .At each point $\alpha(s)$ of curve α ,the planes spanned by {T,N } ,{T,B } and {N,B } are known respectively as the osculating plane ,the rectifying plane and the normal plane [1].The curves $\alpha : I \subset R \to R^3$ for which the position vector α always lie in their rectifying plane ,are simplicity called rectifying curves .Similarly ,the curves for which the position vector α always lie in their osculating plane , are for simplicity called osculating curves. Moreover , the curves for which the position vector α always lie in their normal plane , are for simplicitycalled normal curves [23]. The Minkowski 3-space E_1^3 is the Euclidean 3-space E^3 provided with the standard flat metric given by $g = -dx_1^2 + dx_2^2 + dx_3^2$,Where (x_1 , x_2 , x_3) is a rectangular coordinate system of E_1^3 .Since g is an indefinite metric ,recall that a vector $v \in E_1^3$ can have one of three Lorentzian causal characters ,it can be space like if $g(v,v) > 0$or $v = 0$,Time like if $g(v,v) < 0$ and light like if $g(v,v) = 0$ and $v \neq 0$ .Similarly, an arbitrary curve $\alpha = \alpha(s)$ in E_1^3 can locally be space like ,time like or light like ,if all of its velocity vectors $\alpha'(s)$ are respectively , space like ,time like or light like respectively [I, 24].Minkowski space is originally from the relativity in physics .In fact ,a time like curve corresponds to the path of an observer moving at less than the speed of light , a light like curves correspond to moving at the speed of light and a space like curves moving faster than light [25]. Consider the moving Frenet frame {T,N,B} along the curve $\alpha(s)$ in E_1^3.For an arbitrary curve $\alpha(s)$ in the space E_1^3 ,the following Frenet formula are given in [26].A curve in Lorentzian space L^n is a smooth map $\alpha : I \to L^n$ where Iis the open interval in the real line R .The interval Ihas a coordinate system consisting of the identity map u of I .The velocity of α at $t \in I$ is $\alpha' = \frac{d\alpha(u)}{du}\Big|_t$ .A curve α is said to be regular if $\alpha'(t)$ does not vanish for all t in I. $\alpha \in L^n$ is space like if its velocity vectors $\alpha'$are space like for all $t \in I$,similarly for time like and null. If α is a null curve ,we can reparametrize it such that $<\alpha'(t) .\alpha'(t)>= 0and\alpha'(t) \neq 0$[27]. If α is time like curve ,then the Frent formula read

$$\begin{pmatrix} T' \\ N' \\ B' \end{pmatrix}=\begin{pmatrix} 0 & K_1 & 0 \\ K_1 & 0 & K_2 \\ 0 & -K_2 & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix}$$

Where $g(T,T) = -1, g(N,N) = 1$, $g(B,B) = -1, g(T,N) = 0$, $g(T,B) = 0$and $g(N,B) = 0$.

Definitions

We adapted these definitions and concepts to fit coherently into the framework of this article.

i-A subset A of a topological space X is called a retract of X if there exists a continuous map $r : X \longrightarrow A$ called a retraction such that $r(a) = a$ for any $a \in A$[2,4 5,6].

ii- A subset A of a topological space X is a deformation retracts of X if there exists a retraction $r : X \longrightarrow A$ and a homotopy $\varphi : X \times I \longrightarrow X$ such that:

$$\left.\begin{aligned} \varphi(x,0) &= x \\ \varphi(x,1) &= r(x) \end{aligned}\right\} \quad x \in X$$

$\varphi(a,t) = a,\ a \in A,\ t \in [0,1]$ [3,5] .

Main Results

Let α(s) be a time like helix .Then we can write its position vector as follows:

$\alpha(S) = \lambda(s)T(s) + \mu(s)N(s) + \upsilon(s)B(s)$ ---------(1)

Differentiating equation (1) with respect to s and using Frenet formula we have

$$\lambda'(s) + \mu(s)K_{1=}1 ,\ \lambda(s)k_1 + \mu'(s) - \nu(s)K_2 = 0,\ \mu((s)K_2 + \nu'(s) = 0 \quad \text{----} \ (2)$$

From equation (2) we have

$\mu'' - \mu(s)(K_1^2 - K_2^2\ ) + K_1 = 0$ ----------(3)

Now we are going to discuss the following case

If $K_1^2 < K_2^2$.The solution of equation (3) is

$\mu(s) = C_1 \cos(S\sqrt{K_2^2 - K_1^2}) + C_2 \sin(S\sqrt{K_2^2 - K_1^2}) - \frac{K_1}{K_2^2 - K_1^2}$ -----(4)

Where $C_1, C_2 \in R$..From equation (2) we have $\lambda'(s) = 1 - \mu(s)K_1$.By using (4) we find the solution of this equation as follows:

$$\lambda(s) = S - \frac{C_1K_1}{\sqrt{K_2^2 - K_1^2}}\sin(S\sqrt{K_2^2 - K_1^2}) + \frac{C_2K_1}{\sqrt{K_2^2 - K_1^2}}\cos(S\sqrt{K_2^2 - K_1^2}) + \frac{K_1^2S}{K_2^2 - K_1^2} \quad \text{------}(5)$$

Also from (2) we have $\nu'(s) = -\mu((s)K_2$ .By using (4) we get

$$\nu(s) = \frac{-C_1K_2}{\sqrt{K_2^2 - K_1^2}}\sin(S\sqrt{K_2^2 - K_1^2}) + \frac{C_2K_2}{\sqrt{K_2^2 - K_1^2}}\cos(S\sqrt{K_2^2 - K_1^2}) + \frac{K_1\,K_2S}{K_2^2 - K_1^2}\,. \quad \text{----------}(6)$$

Hence, the position vector $\alpha_2(s)$ is

$$\alpha_2(s) = (\ S - \frac{C_1K_1}{\sqrt{K_2^2 - K_1^2}}\sin(S\sqrt{K_2^2 - K_1^2}) + \frac{C_2K_1}{\sqrt{K_2^2 - K_1^2}}\cos(S\sqrt{K_2^2 - K_1^2}) + \frac{K_1^2S}{K_2^2 - K_1^2})\ T(s) + (C_1 \cos(S\sqrt{K_2^2 - K_1^2}) + C_2 \sin(S\sqrt{K_2^2 - K_1^2}) - \frac{K_1}{K_2^2 - K_1^2})\ N(s) +$$

$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$
$\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$

Now we are going to discuss the retraction of the position vector $\alpha_2(s)$ as follows
Let r_i :$\{\alpha_2(s)-\beta\}\rightarrow\{\alpha_2(s)-\beta\}^*$,where $\{\alpha_2(s)-\beta\}$ be the open helix and $\{\alpha_2(s)-\beta\}^*$ be the retraction of the position vector $\alpha_2(s)$.
Now we discuss the following cases:
If C_1=0,we have the following retraction defined as:
$r_1(\alpha_2(s))=(\,S+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\;T(s)$
$+(\,C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\;N(s)+(\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$
$\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$
Also ,if C_2=0 we obtain the following retraction defined by:
$r_2(\alpha_2(s))=(\,S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\;T(s)\;+(C_1\cos(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\;N(s)+(\,\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})$
$+\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$
If $K_1\;=0$,we present the following retraction given by $r_3\big(\alpha_2(s)\big)=(\,S)\;T(s)\;+(C_1\cos(S\,K_2)+C_2\sin(S\,K_2))\;N(s)+$
$(-C_1\sin(S\,K_2)+C_2\cos(S\,K_2)+)\;B(s).$
If $K_2=0$ we have the following retraction defined as:
$r_4(\alpha_2(s))=(\,S-\frac{C_1K_1}{\sqrt{-K_1^2}}\sin(S\sqrt{-K_1^2})+\frac{C_2K_1}{\sqrt{-K_1^2}}\cos(S\sqrt{-K_1^2})\text{-}\;S\,)T(s)\;+(C_1\cos(S\sqrt{-K_1^2})+C_2\sin(S\sqrt{-K_1^2})+\frac{1}{K_1})\;N(s).$
If $T(s)=0$, we present the following retraction given by:
$r_5(\alpha_2(s))=(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\;N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$
Also, if $N(s)$ =0 we obtain the following retraction defined as:
$r_6(\alpha_2(s))=(\,S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\;T(s)\quad+\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+$
$\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$
Now ,if $B(s)=0$ we present the following retraction given by:
$r_7(\alpha_2(s))=(\,S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\;T(s)$
$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\;N(s).$
Also ,if $T(s)=N(s)$ =0, we have the following retraction defined as:
$r_8(\alpha_2(s))=(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$
$\frac{K_1\,K_2S}{K_2^2-K_1^2})\;B(s).$
Now ,if $T(s)=B(s)=0$ we present the following retraction given by:
$r_9(\alpha_2(s))=(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\;N(s).$
Also ,if $N(s)=B(s)=0$, we have the following retraction defined as:
$r_{10}(\alpha_2(s))=(\,S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\;T(s).$
Moreover ,if $T(s)=N(s)=B(s)$ =0 , we have the following minimal retraction defined as:
$r_{11}(\alpha_2(s))=\{\,0,0,0\}.$

In this position ,we present some cases of deformation retract of time like helix in Minkowski 3-space .The deformation retract of time like helix is

φ: $\{\alpha_2(s)-\beta\}\times I \to \{\alpha_2(s)-\beta\}$,

where $\{\alpha_2(s)-\beta\}$ be the open helix of the position vector $\alpha_2(s)$ and I is the closed interval [0,1],be present as

$$\varphi(x,h): \Big(\Big(\Big(S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)$$
$$+\Big(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+$$
$$\Big(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big)-\beta\Big)\times I \quad\to$$
$$\Big(\Big(\Big(S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)+\Big(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+$$
$$\Big(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big)-\beta\Big).$$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_1(\alpha_2(s))$ is

$$\varphi(x,h)=(1-h)\Big(\Big(\Big(S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)+\Big(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+$$
$$\Big(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big)-\beta\Big)+h\Big(\Big(S+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)$$
$$+\Big(C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+\Big(\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big),$$

Where

$$\varphi(x,0)=\Big(\Big(\Big(S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)$$
$$+\Big(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+$$
$$\Big(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big)-\beta\Big),$$

and

$$\varphi(x,1)=\Big(\Big(S+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)$$
$$+\Big(C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+\Big(\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big),$$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_2(\alpha_2(s))$ is defined as

$$\varphi(x,h)=\quad\cos\frac{\pi h}{2}\Big(\Big(\Big(S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1^2S}{K_2^2-K_1^2}\Big)T(s)+\Big(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2}\Big)N(s)+$$
$$\Big(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{K_1K_2S}{K_2^2-K_1^2}\Big)B(s)\Big)-\beta\Big)$$

$+\sin\frac{\pi h}{2}((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)+(C_1\cos(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})$
$+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s)).$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_3(\alpha_2(s))$ is defined by

$\varphi(x,h)=\quad\frac{1-h}{1+h}(((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)$
$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s))\text{-}\beta\quad)+\frac{2h}{1+h}\quad((S)\,T(s)$
$+(C_1\cos(SK_2)+C_2\sin(SK_2))\,N(s)+$
$(-C_1\sin(SK_2)+C_2\cos(SK_2)+)\,B(s)).$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_4(\alpha_2(s))$ is given by

$\varphi(x,h)=\quad\cos\frac{\pi h}{2}(((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)$
$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s))-\beta)+\sin\frac{\pi h}{2}\;((S-\frac{C_1K_1}{\sqrt{-K_1^2}}\sin(S\sqrt{-}K_1^2)+$
$\frac{C_2K_1}{\sqrt{-K_1^2}}\cos(S\sqrt{-}K_1^2)\text{-}S)\,T(s)+(C_1\cos(S\sqrt{-}K_1^2)+C_2\sin(S\sqrt{-}K_1^2)+\frac{1}{K_1})\,N(s)).$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_5(\alpha_2(s))$ is defined as

$\varphi(x,h)=\quad\frac{1-h}{1+h}(((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)$
$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s))-$
$\beta)+\frac{2h}{1+h}((C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s)).$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_6(\alpha_2(s))$is $\varphi(x,h)=(1-h)(((S-$
$\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)\quad+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-$
$\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s))\text{-}\beta)h((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})\beta)+$
$\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)+\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s)).$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_7(\alpha_2(s))$ is defined as

$\varphi(x,h)=e^h(1-h)(((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)$
$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s)+$
$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1K_2S}{K_2^2-K_1^2})\,B(s))-\beta)+$
$\frac{h}{2}\left(2h+\frac{1}{2}\right)((S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\,T(s)\;+(C_1\cos(S\sqrt{K_2^2-K_1^2})+$
$C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\,N(s))\,.$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_8(\alpha_2(s))$ is defined by

$$\varphi(x,h) = \frac{1-h}{1+h}\ (((\ S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\ T(s)$$
$$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\ N(s)+$$
$$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\ B(s))-\beta)+\frac{2h}{1+h}((\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\ B(s)).$$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_9(\alpha_2(s))$ is defined as

$$\varphi(x,h) = \ln e^{(1-h)}(((\ S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\ T(s)$$
$$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\ N(s)+$$
$$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\ B(s))-\beta)+$$
$$\ln e^{h}((C_1\cos(S\sqrt{K_2^2-K_1^2})+\ C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\ N(s)).$$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_{10}(\alpha_2(s))$ is defined as

$$\varphi(x,h) = e^{h}(1-h)(((\ S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\ T(s)$$
$$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\ N(s)+$$
$$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\ B(s))-\beta)+\frac{h}{2}\left(2h+\frac{1}{2}\right)((\ S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+$$
$$\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\ T(s)).$$

The deformation retract of the helix $\alpha_2(s)$ into the retraction $r_{11}(\alpha_2(s))$ is defined as

$$\varphi(x,h) = \ln e^{(1-h)}\ (((\ S-\frac{C_1K_1}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_1}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1^2S}{K_2^2-K_1^2})\ T(s)$$
$$+(C_1\cos(S\sqrt{K_2^2-K_1^2})+C_2\sin(S\sqrt{K_2^2-K_1^2})-\frac{K_1}{K_2^2-K_1^2})\ N(s)+$$
$$(\frac{-C_1K_2}{\sqrt{K_2^2-K_1^2}}\sin(S\sqrt{K_2^2-K_1^2})+\frac{C_2K_2}{\sqrt{K_2^2-K_1^2}}\cos(S\sqrt{K_2^2-K_1^2})+\frac{K_1\,K_2S}{K_2^2-K_1^2})\ B(s))-\beta)+\ln e^{h}\{\ 0,0,0\}.$$

Conclusion

Consider a curve in a space suppose that the curve is sufficiently smooth so that the Frent frame adapted to is defined the curvature k_1 and torsion $k_2$then provide a complete characterization of the curve .

Helix is one of the most fascinating curves in Science and Nature ,a helix is a geometry curve with non – vanishing constant curvature k_1 and non-vanishing constant torsion k_2.A curve of constant slope or general helix in Euclidean 3-space E^3 ,is defined by the property that the tangent makes a constant angle with a fixed straight line .

In the present article, we obtain and study types of retraction of the position vectors of time like helix in Minkowski 3-space.Also ,by using the position vectors of the curve and retraction of the position vectors ,we deduced types of the deformation retracts of the time like helix in Minkowski 3-space. The relations between the retraction and the deformation retracts of time like helix are obtained. Types of minimal retraction of time like helix in Minkowski 3-space are also presented. New types of homotopy maps are deduced .

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