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Optimization of multi-objective portfolio using imperialist competitive algorithm in Tehran Stock Exchange

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Abstract: *Most real-world optimization has several objectives that generally conflict with each other. Investors in the capital markets pursue several objectives to optimize stock portfolio as well. The present study aimed to optimize multi-objective portfolio using imperialist competitive algorithm in Tehran Stock Exchange. The statistical sample included the top 30 companies listed in Tehran Stock Exchange (TSE) from 2005 to 2015. For this aim, we first used an autoregressive integrated moving average (ARIMA) model to model the return series. Then, portfolio risk was firstly calculated based on generalized autoregressive conditional heteroscedasticity (GARCH) variance models in compliance with the Markowitz approach. Moreover, the results obtained from the present study showed that the imperialist competitive algorithm works well in developing stock portfolios. According to the findings, it is confirmed and recommended to apply the imperialist competitive algorithm for selecting and optimizing stock portfolios. The successful performance of this algorithm is due to the continuous superiority over the certificate market portfolio to the claim of their compatibility with the problem, which is not negligible and deniable.*

Keywords: *Risk, Return, Portfolio, Asset Portfolio, Imperialist Competitive Algorithm*

INTRODUCTION

The existence of an active and prosperous capital market is known as a factor of countries' development. In such countries, most investments are made through financial markets, and the active participation of people in the stock exchange will ensure maintenance of the capital market and sustainable development of the country. In the active participation of people in capital markets, the main problem facing each investor is to pick the right stocks and develop an optimal portfolio. Hence, widespread efforts are done by investigative researchers to propose stock analysis methods and improve them. Efforts to improve stock analysis methods (especially in markets with high diversified stocks) have led to emergence of new methods that, along with the previous approaches, seek to maximize profits in financial markets (Gupta, P., Mehlawat, M.K., Saxena, 2008).
The main objective of portfolio selection problems is to compute a portfolio from a given portfolio of assets that minimizes risk for a minimum level of expected returns. This can be evaluated by the portfolio variance. This approach is important because of offering portfolio diversification as an investment criterion rather than focusing on maximizing returns as the only parameter. In fact, the base mode of this problem is a quadratic model that can be solved analytically using standard tools (Szego, 2011).
In the single-period portfolio problem, it is assumed that the investor decided to allocate the N existing assets once at the beginning of the desired period based on the risk and the relationships between returns during the investment horizon. Decisions are made only once, and there is no permission to review until the end of the

period and the effect of decisions on subsequent periods is not taken into consideration. Multi-period portfolio optimization is considered as a dynamic multi-period problem so that transactions occur at discrete time points. Multi-period portfolio optimization problem can be defined as follows: N risky assets, a risk-free asset, the planning horizon including T time steps, in which decisions are made (t = 0, 1, ..., T-1). Time period is the time intervals between two steps. These time intervals range from one minute to several years and decisions are made at the beginning of each step. The first step represents the present i.e. the zero moment.

Sales proceeds are added to cash (zero asset) and purchasing costs subtracted from cash. At the moment t + 1, the investor asset is updated and borrowed based on the returns obtained within the time interval [t, t + 1]. For simplicity, it is assumed that returns on the risk-free asset are constant for lending and borrowing. Date of the time horizon is determined based on the critical constraints of the investor such as date of paying off a large debt, and we focus on the investor's status at the end of the T period. At the end of the investment period T, the investor collects his ultimate wealth. His main goal is to manage a portfolio in such a way that the optimality of his ultimate wealth be maximized (Shen, R., Zhang, 2015).

Therefore, portfolio selection problem considering constraints is of great importance. So far, various innovative methods are introduced to optimize portfolios and their results are presented. Thus, considering the importance of portfolio optimization and the aforementioned questions, the present study aimed to optimize portfolios using the imperialist competitive algorithm and reviewing the research conducted using other techniques and using the game theory to determine the optimal portfolio of assets with a multi-objective approach. This study determines that to achieve an optimal portfolio, how much and which stock should contribute to the portfolio development.

The present paper consists of five sections. In the next section, the literature review is presented. The third section deals with methodology. The experimental model is presented in the fourth section. Finally, the final section is conclusions and recommendations.

2. Literature review

Risk and returns are two of the key elements in making investing decisions in stock markets. Investors seek to increase return on one hand and reduce risk on the other hand. Most investors prefer confidentiality to uncertainty, so they rely on a certain level of return for risk reduction. This confidentiality is possible through diversification and portfolio development. Portfolio is a collection of stocks in which each stock has certain risk and returns. In capital markets, various methods and techniques are used for this aim. Portfolio optimization problems have been considered by researchers since early 1952. The new portfolio theory, first proposed by Markowitz, created an organized paradigm to develop a portfolio with the highest expected return rate at a certain level of risk (the property of all portfolios in an efficient collection). According to Markowitz's theory, for a certain level of return, one can minimize the portfolio variance by minimizing investment risk or, at a certain level of risk that is tolerable to the investor, one can consider the maximum return that increases portfolio's expected return rate (Lin et al., 2007).

The Markowitz model can be solved using mathematical programming models, but when the real-world constraints such as large number of investments, stock weight limits, etc. are added to it, the search space becomes very large and discontinuous, which makes it virtually impossible to use mathematical models. Therefore, the need for nonlinear patterns and models to determine stock behaviors has a significant impact on stock prediction and proper decision making. Recently, new innovative methods are applied to solve these problems.

Like other evolutionary algorithms, the imperialist competitive algorithm also starts with a number of primary random populations, each of which is called a "country". Some of the best members of the population are chosen as imperialists. The rest of the population are considered as colonies. The colonialists, depending on their power, attract the colonies based on a particular process. The total power of each empire depends on both its constituent parts i.e. the imperialist country (as the core) and its colonies. In mathematical terms, this dependence is modeled by defining power of the empire as the power of the imperialist country plus a percentage of the average power of its colonies.

With the formation of primitive empires, imperialist rivalry starts. Any empire that failed in imperialist competition and did not increase its power (or at least avoid its influence reduction) will be eliminated from the scene of imperialist competition. Therefore, the survival of an empire depends on its power to attract the competing empire colonies and dominate them. Consequently, during the imperialist rivalries, the power of

larger empires will gradually be increased and weaker empires will be eliminated. Empires will have to develop their colonies to increase their power.
Over time, colonies will become closer to empires in power and a kind of convergence will be seen. The ultimate colonial rivalry is when we have a single empire in the world with colonies very close to the imperialist country in terms of situation.
In optimization, the goal is to find an optimal solution in terms of problem variables. An array of the variables that should be optimized is created. In the genetic algorithm, this array is called "chromosome". Here, it is called "country". In a n-dimensional optimization problem, a country is a n × 1 array:

$$\text{Country} = [\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_{\mathrm{Nvar}}]$$

In fact, in solving an optimization problem using the introduced algorithm, we are looking for the best country (the country with the best social-political characteristics). Finding this country is, in fact, equivalent to finding the best problem parameters resulting in the minimum cost function. The cost of a country, like genetics and the PSO, is determined by an assessment function. First, a number of countries are generated, then, based on their cost estimation by the assessment function, some of them are chosen as empires and the rest of them are selected as colonies (the number of empires, colonies and primary countries, which are the problem parameters, is determined by the user). Then, colonies are distributed among them using random operators and taking into account the costs of the empires. Given the primitive state of all empires and distribution of colonies, the imperialist competitive algorithm starts. The evolution process is located within a loop that continues until a stop condition is fulfilled. To start the algorithm, we generate N primary countries ($N_{country}$). We chose N_{imp} best members of this population (the countries with the minimum cost function) as imperialists. The rest N_{col} countries are the colonies, each of which belongs to an empire. In order to distribute the initial colonies among the imperialists, we give a number of colonies to each imperialist based on its power. To do this, given the costs of all imperialists, we consider their normalized costs as follows:

$$C_n = mac\{c_i\} - c_n$$

Where, c_n is the cost of the imperialist n, $mac\{c_i\}$ is the maximum cost among the imperialists, and c_n is the normalized cost of this imperialist. Any imperialist with higher cost (weaker imperialist) will have a lower normalization rate. Given the normalized costs, the normalized relative power of each imperialist is calculated as follows and, accordingly, colonial countries are distributed among imperialists.

$$P_n = \left| \frac{c_n}{\sum_{i=1}^{N_{imp}} c_i} \right|$$

From another point of view, the normalized power of an imperialist is the ratio of colonies governed by that imperialist. Therefore, the initial number of colonies governed by an imperialist will be equal to:

$$N.c_n = rand\{p_n, (N_{col})\}$$

Where, $N.c_n$ is the initial number of colonies belonging to an empire, N_{col} is the total number of colonial states in the primary countries population, and rand is the function that gives the closest integer to a decimal number. Given $N.c_n$ for each empire, we will randomly select $N.c_n$ countries from the primary colonies and give them to the imperialist n. Given the initial state of all empires, the imperialist competitive algorithm starts. The evolution process is located within a loop that continues until a stop condition is fulfilled.
According to the aforementioned explanations, the fitness function is defined as follows:

$$f_{cost} = \lambda \left[\sum_{i=1}^{n} \sum_{j=1}^{m} z_{pi} x_{pi} z_{pj} x_{pj} \sigma_{ij} \right] - (1-\lambda) \left[\sum_{i=1}^{n} z_{pi} x_{pi} \mu_i \right]$$

Where, f_{cost} represents the fitted value of the particle or country p. In order to apply limits on the number of selected stocks, we define variable $k_p^* = \sum_{i=1}^{n} z_{pi}$ and Q set. Q is the portfolio containing the particle (portfolio) P and k_p^* represents the number of stocks in the Q set. If k is assumed to be the desired number of assets in the portfolio, when $k_p^* > k$, a number of stocks should be excluded from Q until $k_p^* = k$. To decide which stock should be added to or excluded from the Q set, the relative effect of each stock on the fitness function (C_i) is measured. Stocks or assets whose relative effect on the fitness function is high, are preferred to be added to the Q set, and on the contrary, the stocks with less elative effect on the fitness function are preferred to be excluded from the Q set. C_i is calculated from the following equations:

$$\theta = 1 + (1-\lambda)\mu_i, \quad i = 1, \dots, N$$

$$\rho_i = 1 + \lambda \frac{\sum_{i,j=1}^{N} \sigma_{ij}}{N}, \quad i,j = 1, \dots, N$$

$$\Omega = -1 \times \min(0, \theta_1, \dots, \theta_N)$$

$$\psi = \min(0, \rho_1, \dots, \rho_N)$$

$$C_i = \frac{\theta_i + \Omega}{\rho_i + \psi}, \qquad i = 1, \dots, N$$

Thus when $k_p^* > k$, the stock with the minimum c value is excluded from the Q set, and when $k_p^* < k$, the stock with the maximum c value is added to the Q set as mentioned, the dimensions x_i in each particle show the ratio of capitalization. The sum of the dimensions x_i of the stocks within the Q set must be equal to one. If $\aleph$ is the sum of x_is, using the $x_{pi} = \frac{x_{pi}}{\aleph}$ transform for all Q-member stocks, the limit of the equation 9 is satisfied. The limit $\varepsilon_i \le x_i \le \delta_i$ must also be satisfied for all Q-member stocks. To apply this limit, the variables $t_i = \delta_i - z$ and $e_i = x_{pi} - \varepsilon_i$ are defined for the Q-member stocks. δ^* is the sum of t_is and ε^* is the sum of e_is. η is the sum of $t_i \times 1$ for $t_i < 0$. φ is the sum of $e_i \times 1$ for $e_i < 0$.

Tukat et al. (2003) analyzed the multi-step asset allocation problem under normal and stable returns scenarios. They used random planning and decision rules to solve this problem. Using econometric models, e.g. ARMA and GARCH, they generalized their extracted models of return and asset fluctuations and estimated future returns and volatilities. They concluded that modeling heavy-tailed distributions can play a very effective role in allocating assets. Maidan et al. (2010) used multi-period portfolio with risk-variance and skewness measures to optimize the electricity market. Sajjadi et al. (2011) presented a fuzzy multi-period portfolio considering different lending and borrowing rates. San et al. (2011) solved the multi-period portfolio optimization problem using a new algorithm named Discrete Particle Swarm Optimization (DPSO) and showed that it performs better than the other algorithms. Sedzer et al. (2012) proposed a process that enables integrated asset allocation based on the hierarchy analysis process, mean-variance model, and ideal planning. They considered scenarios of future economic conditions and investor risk as factors affecting asset allocation. In this approach, the AHP method was used to consider market conditions and investor risk. Weighted values of these factors were used to show their importance in the allocation process. Then, the mean-variance optimization was performed to determine the maximum return for different levels of variance-return. Ultimately, due to the non-uniqueness of this efficient boundary, ideal planning was used to integrate all correlated factors in asset allocation decision making to minimize any downward and upward deviations from the ideal ratios of stocks, securities and cash assets derived from the AHP model. Finally, comparison of the results obtained from returns and variance of this portfolio with S&P/TSX60 index (as the reference portfolio for the Canadian market) indicated that return to standard deviation ratio for this portfolio was 5.597%, while this ratio was 1.070% for the Canadian market portfolio. Pindoria et al. (2014) presented a multi-objective mean-variance-kurtosis model for allocating investment portfolios and showed that when the distribution function of assets is abnormal, their model can significantly create better portfolios. Chang (2015) studied mathematical portfolio optimization models. He stated that portfolio optimization problem is one of the pillars of applied mathematics. The portfolio optimization problem is one of several types of nonlinear multi-objective problems. In financial science, it was always a question that how to combine investments to form an optimal portfolio. This problem is called optimal portfolio selection which dates back to the 1950s. Markovitz's approach for solving the optimal portfolio selection problem with the minimum risk and maximum return is one of the most widely used theories in the financial markets. In this study, the classic Markovitz mean-variance model was first introduced. In order to make this approach more efficient, the idea

of using higher-order torque in the portfolio optimization problem has been raised in recent years. This idea was first proposed by Kono et al. (1990). Considering that if the distribution of product returns around the mean value is asymmetric, the third-order torque plays an important role, and in particular, if the investor can choose between portfolios with equal mean and variances, he prefers the portfolio with higher third-order torque. The difference of multi-objective models with the classic Markovitz model is that investors consider other considerations other than risk and return when creating their portfolios, such as increasing liquidity or reducing borrowing sales, etc. So, there is a randomized multi-objective programming problem that is converted into definitive equivalent problem to be solved.
Najafi and Moushikhan (1393) modeled and presented an optimal solution for optimizing the multi-period portfolio using genetic algorithm. One of the most attractive decision making problems is financial optimization in uncertain situations. The single- period portfolio selection problem is one of the classic financial problems, but this model was based on three constraining assumptions:

1- The investment horizon is short-term;
2- The transaction cost is not considered;
3- The problem parameters are definite and known.

The present study seeks to present and solve a model in order to overcome these constraints and bring the model closer to the real world. Hence, here we propose the multi-period portfolio optimization probability model of mean-half-variance-conditional value at risk considering transaction costs. After modeling, it was solved using a genetic algorithm. In this study, we used 24 stock data collected from the companies listed in Tehran Stock Exchange were used as model inputs from December 2008 to August 2013. The results showed that this algorithm is suitable for solving this class of problems with a good performance. Moushikhan and Najafi (2014) performed portfolio optimization using multi-objective particle swarm algorithm for the multi-period probability model of mean-half-variance-skewness. Financial optimization is one of the most attractive areas for decision making in uncertain conditions. Single-period investment portfolio selection problem is a classic financial problem, but this model was based on three constraining assumptions. Hence, in the present study we first proposed the multi-period portfolio optimization probability model of mean-half-variance-skewness considering transaction costs. Due to the nonlinear nature of the problem, it is very challenging to solve the multi-period investment portfolio problem. Therefore, after modeling the problem using the multi-objective and single-objective particle swarm optimization algorithms, we solved the proposed model. The results showed that multi-objective particle swarm optimization algorithm acts better than single-objective particle swarm optimization algorithm. Darabi et al. (2016) selected the optimal portfolio from companies listed in Tehran Stock Exchange. A portfolio is a mix of assets formed by an investor for investment. Portfolio selection process is one of interesting problems considered by many researchers. Different factors involved in this process have been changed over time which makes it necessary to use an appropriate tool to support investment decisions. The present study aimed to propose an intelligent model for optimal portfolio selection using the improved differential evolution algorithm. To this aim, the risk and expected returns of the companies listed in Tehran Stock Exchange were studied monthly. The statistical sample included financial data of 102 companies listed in Iran's stock market during 2009-2010. The results showed that the proposed model can lead to optimal portfolio selection by considering the interactions between risk and expected returns. Mirzaei et al. (2016) examined the use of multi-objective genetic algorithm for stock portfolio optimization using technical indices. The classic goals of financial science were based on the balance between return and risk and analyzing it at various opportunities was the topic of many financial management research. The use of technical indices is one of portfolio management tools. The present study seeks to use these indices to extract stock trading rules. The time interval of our study was from the beginning of 2009 to the end of 2014 and the sample included 216 companies. During 2009 to 2011, we used technical indices and multi-objective genetic algorithm to maximize return and minimize risk and obtained a model for optimal portfolio management. During 2012 to 2014, this model was used for optimal portfolio management. In order to evaluate this model, the results were compared with Tehran Stock Exchange Index and it was shown that using technical indices, we can have a better performance in the market.

3. Methodology

This study was an applied research in terms of objective because it aimed to examine a particular model in the capital market. The research methodology was descriptive and correlational. Using statistical methods and econometrics, in the first step, we examined optimal portfolio using the mean and variance predicted for each asset using imperialist competitive algorithm. Data needed for capital distribution among different groups of stocks were presented as the time series of three-month stock prices of the top 30 companies listed in Tehran Stock Exchange.

Portfolio optimization is the choice of the best combination of financial assets in such a way that the return of investment portfolio be maximized and the risk be minimized. The basic idea of the modern portfolio theory is that if we invest in the assets that are not completely correlated, the risks neutralize each other, so a steady return interval can be obtained at a lower risk. Markowitz (1952) proposed an algorithm to solve the optimal portfolio selection problem (the mean variance theory) for the first time. He presented the problem as quadratic planning for minimizing variance of assets so that that the expected return be equal to a constant value.

Risk aversion of all investors is the main assumption of this model. This problem has another functional limitation, according to which the sum of asset weights should be equal to one. Moreover, the weight of each asset in a portfolio should be a real non-negative number. The standard form of the mean variance model is as follows:

$$
\begin{aligned}
& Min \sum_{i=1}^{n}\sum_{j=1}^{m} x_i x_j \sigma_{ij} \\
& \text{Subject to } \textstyle\sum_{i=1}^{n} x_i u_i = R^* \\
& x_i \geq 0 \ (i = 1, \dots, n) \\
& x_j \geq 0 \ (j = 1, \dots, m)
\end{aligned}
\tag{1}
$$

Fernández and Gomes modified the Markowitz model by adding upper and lower limits for variables, and created the CCMV model or the mean variance model with bounded components. The general form of this model is as follows:

$$
\begin{aligned}
& Min \sum_{i=1}^{n}\sum_{j=1}^{m} x_i x_j \sigma_{ij} \\
& \text{Subject to } \textstyle\sum_{i=1}^{n} x_i u_i = R^* \\
& \sum_{i=1}^{n} x_i = 1 \\
& \varepsilon_i \leq x_i \leq \delta_i \\
& x_i \geq 0 \ (i = 1, \dots, n) \\
& x_j \geq 0 \ (j = 1, \dots, m)
\end{aligned}
\tag{2}
$$

Where, ε_i and δ_i are the lower and upper limits of the variable i (contribution of the stock i in the portfolio), respectively. If the limit on the number of selected assets is added to the problem, the model will be as follows:

$$Min\ \lambda \left[\sum_{i=1}^{n}\sum_{j=1}^{m} z_i x_i z_j x_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{n} z_i x_i \mu_i\right] \quad (3)$$

(4)

$$\text{Subject to } \textstyle\sum_{i=1}^{n} x_i = 1 \quad (5)$$

(6)

$$\sum_{i=1}^{n} z_i = k \quad (7)$$

(8)

$$\varepsilon_i z_i \leq x_i \leq \delta_i z_i$$

$$z_i \epsilon [0,1] \ (i = 1, \dots, n)$$
$$x_j \geq 0 \ (j = 1, \dots, m), x_i \geq 0 \ (i = 1, \dots, n)$$

In the above mathematical model, λ varies within the interval [0,1] so that if λ=0, the total weighting coefficient is assigned to the return and the portfolio with maximum return is selected regardless of risk. If λ=1, the total weighting coefficient is assigned to the risk and the portfolio with minimum risk is selected regardless of return. For 0< λ <1, portfolios are optimized by taking into account both risk and return factors. In other words, with increasing λ, it is more important to reduce risk and since (1-λ) is decreased, so maximizing return becomes less important. z_i is the variable of decision about investment in each stock. If z_i=1, then i will be in the portfolio. According to the second limit, the total number of stocks in the portfolio is equal to K and ε_i and δ_i are the lower and upper limits of the variable i (contribution of the stock i in the portfolio), respectively. The CCMV system of equations is a combination of the integer programming problem and the quadratic programming problem. There are no efficient algorithms in mathematical programming to solve these problems. In this paper, in order to create the optimal portfolio, the optimal portfolio was examined by the imperialist competitive algorithm.
The main model used in this study was the imperialist competitive algorithm (ICA). This study was an applied research in terms of objective and data used to perform this research were collected from 30 large companies listed in Tehran Stock Exchange during 2010-2015. The main variables of the research were stock selection and determination of its contribution in the portfolio set. In this study, each imperialist represented a portfolio and the countries with the best position made up the investment efficient boundary. N×2 dimension is considered for each country where N is the total number of assets. The first N dimension is related to the investment ratio per share (x_{pi}) and the second N dimension includes investment decision making variables (z_{pi}). P=1, …., N indicates the country number in the portfolio, and P is the total number of countries. i=1,…,N represents the stock number in the country.

4. Experimental model

4-1 Statistical characteristics of research variables

Since the performance of different time series models can be affected by different data, first we examined descriptive statistics of daily return data collected from 30 large companies listed in Tehran Stock Exchange from April 5, 2010 to March 1, 2015, presented in Table1:

Table 1: Descriptive statistics of return series during the case study period

Statistics	Return series of company	Return series of market
Mean	0.294	0.097
Max	5385.07	5.400
Min	-118.270	-5.510
S.D	0.893	1.283
Skewness	276.415	0.406
Kurtosis	11.866	7.709
Jarque- Bra	198.195 (0.0000)	199037 (0.0000)

Source: Research findings

The above table shows that mean values of the company and market return series during the desired period were equal to 0.294 and 0.097, respectively. Moreover, the standard deviations of the company and market return series were equal to 0.893 and 1.283, respectively. By comparing these two, one can observe many variations in the series. Normality test of the given series distribution (Jarck-Bra test) also indicates the abnormality of the probability density distribution function of these series. Skewness and kurtosis coefficients represent right-skewed series with higher kurtosis than a normal distribution.

4-2 Unit root, autoregressive and heteroscedasticity variance tests

In this section, in order to prevent false regression in the model, we investigated the unit root in the research variable. In traditional econometric methods, to test stability of variables, it is assumed that then variables

are stable. In most cases, stability hypothesis is tested using instability and the unit root of the series (autoregressive series).
ADF is one of unit root tests for which the null hypothesis implies the existence of unit root in the variable. Disadvantage of this test and the similar tests is that most tests have a low power to test stability, so null hypothesis is usually accepted and in most cases, this conventional approach rejects stability of series by mistake.
To this aim, in this section, we performed the unit root test using ADF and Phillips Paron (PP) tests in which the null hypothesis indicates the existence of unit root and instability of the variable. We also performed KPSS test, which has a high power in detecting unit root and the null hypothesis in this test implies a lack of unit root and stability of the variable. As shown in Table 2, according to the results obtained from Augmented Dickey Fuller (ADF) and Phillips Paron (PP) unit root tests, the company and market return indices were less than 0.05 at the significant level of 95%. So the null hypothesis was rejected and variables were stable. The following table presents the results of these tests:

Table 2: Statistics of the unit root, autoregressive and heteroscedasticity variance tests performed for series during the case study period

Statistics	Daily return series of company	Daily return series of market
ADF	5221.82 (0.0000)	3630.80 (0.0000)
Phillips-Perron	7901.72 (0.0000)	7907.53 (0.0000)
Box- Ljung Q(10)	488.24 (0.0000)	8643.41 (0.0000)
McLeod-Li $Q^2(10)$	254.21 (0.0000)	89.55 (0.0000)
ARCH (10)	15.32 (0.0000)	12.18 (0.0000)

Source: Research findings

The above table shows that Box-Liung statistic (with ten lag times) for returns rejects the null hypothesis of "the absence of serial autoregressive between terms of the series " and high values of this statistic indicates autoregressive between different lags of this series. Furthermore, McLeod-Li statistic also rejects the null hypothesis of "the absence of serial autoregressive between square returns" which in fact indicates non-linear effects in this series and also confirms its heteroskedasticity variance. It is worth mentioning that the results of the ARCH test were in consistent with the results of the McLeod-Li test and confirm the hypothesis of heteroskedasticity variance of return series.

4-3 Modeling stock returns of companies
Given stability of return series of companies, the best model of the family of ARMA (p, q) models was estimated using the Box-Jenkins methodology to explain behavior of the first-order torque of the above series (mean equation). Therefore, based on the number of significant clusters of correlogram and examining out Layers (by introducing them as dummy variable), the best models were estimated. Table 3 shows the results of the best possible models:

Table 3: Various types of ARMA models for stock return series of companies

Criterion / Model	SBC	AIC
ARMA(1,1)	-11.41	-12.45
* ARMA(1,2)	-11.43	-12.46
ARMA(2,2)	-11.42	-12.43

Source: Research findings

According to the above table, it can be seen that the ARMA (1,2) has the most ideal conditions among the other models, because it has the lowest data criteria (Akaike and Schwarz). Its explicit form is as follows:

$$RET = 0.0008 - 0.035AR(1) + 0.153MA(1) - 0.185MA(2)$$
$$t: \quad (2.11) \quad (-3.24) \quad (2.35) \quad (-21.92)$$
$$R^2 : 0.45 \qquad F - Statistic : 54.65(0.000) \quad DW : 1.89 \quad Log - Likelihood : 453.34$$

Regarding the existence of heteroskedasticity variance in the series, to estimate the suitable mean equation in which significance of coefficients is not ambiguous (or not to lose the significance of statistics t, F, etc.), we used robust regression. Given the fact that in this method the standard error is estimated correctly (hence, it is not necessary to know the explicit form of variance equation), there will no concern about this.

4-4 Estimation of autoregressive conditional heteroskedasticity variance model

As previously stated, the returns series has conditional heteroskedasticity variance which leads to consequences such as:

1) Estimates, though torqueless, are no longer effective.
2) The variance of errors and the variance of coefficients are not torqueless.
3) F and t statistics are highly misleading.

Due to the fact that in the ARIMA models, variance of components is assumed to be constant, which is not true in reality, GARCH models can be used to fix this restriction. In this regard, it is necessary to mention the following tips:

1- The results of the Jarck test (Table 1) rejects the null hypothesis of normality of the probability density distribution function of return series. So, normal distribution should not be used to estimate GARCH models.
2- The heavy-tailed probability distribution function of most financial assets returns indicates the use of t distribution instead of normal distribution to estimate various GARCH models. Because in heavy-tailed data, higher weights are given to the tails and it is more appropriate and more precise to use t Student distribution for estimation of GARCH models.
3- Comparison of different types of GARCH models with different mean equations (e.g. ARMA (2,1), ARMA (2,2), ARMA (3,3), ...) based on t Student distribution and according to Akaike-Schwarz criteria shows that none of them led to good results. Whereas, a composition of GARCH models with the ARMA (1,2) mean model led to the best results. Thus, Table 4 shows the results of estimating the best possible models:

Table 4: Different types of GARCH models with the ARMA (1,2) mean model

Type of model	**ARIMA(1,2)**	
	Akaike statistic	Schwarz statistic
GARCH	18.76	19.87
EGARCH	16.98	17.21
GJR-GARCH	19.73	20.64
APGARCH	18.36	19.87
IGARCH	20.87	21.59

Source: Research findings

According to the above table, it is necessary to point out the following tips:

1- "ARCH in Mean" effects were also examined the modeling of stock price fluctuation series. The results indicated the lack of these effects in the fluctuation series with this mean equation.
2- EGARCH model was the best model in comparison with other possible models.
3- Analysis of Durbin-Watson statistic and LM test for all above models showed that there is no autoregressive between their disturbance components. Furthermore, the Q (Box-Liung) and McLeod-Li tests performed on the residue models rejected autoregressive between disturbance components and squares. So, these results clearly indicate the reliability of estimated models.

Therefore, the final explicit form of EGARCH model with the ARIMA (1,2) mean equation is as follows:

$$RET_t = 0.0008 - 0.035AR(1) + 0.153MA(1) - 0.185MA(2)$$
$$z: \quad (4.45) \quad (-2.95) \quad (2.50) \quad (-3.12)$$
$$h_t = -0.18 + 0.37\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + 0.07\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + 0.93\log h_{t-1}$$
$$z: (-26.39) \quad (19.39) \quad (4.89) \quad (254.12)$$

Table 5: Estimation of EGARCH model for stock return

Mean equation		
Variable	Coefficient	Significance level
Intercept	0.0008	0.000
AR(1)	-0.035	0.007
MA(1)	0.153	0.005
MA(2)	-0.185	0.001
Variance equation		
Intercept	-0.188	0.000
ABS(RESID(-1)/@SQRT(GARCH(-1)))	0.375	0.000
RESID(-1)/@SQRT(GARCH(-1))	0.078	0.000
LOG(GARCH(-1))	0.934	0.000
Goodness of Fit Statistic	Coefficient of determination for Durbin-Watson statistic	0.658 1.93

Source: Research findings

As can be seen from the t statistic presented in the above table, most coefficients of the above model are significant. Moreover, by examining stability of the above model (IGARCH assumption), it can be seen that given that the sum of coefficients of the above model is less than 1, this condition is satisfied and the shocks applied on disturbance components are not stable (IGARCH assumption is rejected). It should be noted that all coefficients of the above variance equation were positive and in consistent with the theory. Also, according to the results obtained from estimated EGARCH model, it was shown that there is an asymmetry between positive and negative shock parts of stock price returns. In the above fitted model, coefficients of the asymmetric part are significant which indicates that even if the model parameters are negative, the variance will be positive. Hence, there is no need for applying any abnormality limit on coefficients. This is possible to take into account the effects of asymmetry of positive and negative shocks on instability and fluctuations of stock prices returns. In other words, the results indicated that positive and negative shocks caused by stock price fluctuations have different effects on stock returns. Finally, it should be noted that goodness of fit statistics including coefficient of determination and Durbin-Watson statistic indicated high explanation power of the model and the lack of autoregressive between disturbance terms of the fitted regression equation. Also, according to the results of the ARCH test, it was found that heteroskedasticity variance problem was also fixed in disturbance terms of the regression model. According to the results, it was observed that because the Prob values reported for all estimated models were higher than 0.05, the null hypothesis of lack of heteroskedasticity variance is not rejected at the significance level of 95%.

The parameters used in the model are reported in Table 6 through the imperialist competitive algorithm for selecting optimal portfolio.

Table 6: Estimation of the imperialist competitive algorithm parameters for selecting optimal portfolio

Total number of colonies	150	Damping factor	0.995
Number of imperialists	30	Empires coefficient of unification	0.04
Colonial revolution rate	0.55	Number of generations	10000
Angle of coefficient of absorption and assimilation of colonies	0.45	Limitation of time lag	Unlimited
Coefficient of influence of colonies on the power of the whole empire	0.04	Limitation of number of generation	Unlimited
Coefficient of absorption and assimilation of colonies	4	Limitation of change in target function	0.000005
Function fitness parameter	25	Number of nodes	10

Table 7 shows the values of the objective function and investment per share.

Table 7: The values of the objective function and investment in the imperialist competitive algorithm
Fitness function=
-1.2434648712e+05
= Matrix solution
[0.01,0.479,0.39,0.91,0.514,0.01,0.48,0.02,0.69,0.18,0.05,0.31 /
2,5,7,10,14,17,18,20,22,24,27,28]

Table 8: The weights assigned to the portfolio forming companies

Company	**Optimal imperialist competitive portfolio**	**Company**	**Optimal imperialist competitive portfolio**
Gulf Petrochemical Industries co.	5.31	Saipa	0
Parsian Oil & Gas Development Group Co.	0	GasPipe company	0
Ghadir Investment co.	4.19	Islamic Republic of Iran Shipping Company	3.29
Mobarakeh Steel Company	2.45	Sepahan oil Co.	1.38
Bandar Abbas Oil Refining Company	3.51	Iran Tractor Industrial Group	0
Mobile communication company	0	Mines and Metals	2.23
Iran Telecommunication Company of Iran	0	Development Investment Company	4.19
Gol-o Gohar Mining and Industrial Company	4.67	Bahman Group	0
Mapna Group	2.21	Pasargad Bank Capital Management Co.	3.23
National Iranian Copper Industries Company	0	Shazand Petrochemical Company	1.13
Esfahan Oil Refining Company	3.32	North Drilling Co.	2.38
Chador Malu Mining and Industrial Company	3.25	Sepahan Oil Co.	1.28
Kharg Petrochemical Company	3.48	Pardis Petrochemical Co.	4.29
.Abadan Petrochemical Co	4.12	Iran Khodro	0
Pars Dara	1.37	Iran Khodro Diesel	0

The values of the objective function resulted from the imperialist competitive algorithm is marked as fitness function. In the solution matrix, the first 10 values represent investment of selected stocks in the optimal solution, the second 10 values indicate the selected stock number in portfolio. According to the table and the

results obtained from assigning weights to 25 companies forming the portfolio, the results are as follows. It should be noted that 5 companies were excluded from the sample of optimal investment portfolio due to reduction in the weight assigned to them. According to the results, one can recommend the use of evolutionary methods, especially multi-objective evolutionary methods for solving portfolio optimization problem better and predicting it. The values presented in the table represent the stocks that must be selected in the portfolio and the amount of each share in the portfolio.
Finally, Table 9 shows the performances of the imperialist competitive algorithm and the traditional return-risk approach based on RVAR or Sharp criterion.

Table 9: The status of algorithms in the formation of a portfolio with input data based on the Sharp criterion

Method	RVAR Performance based on	Performance rank
Imperialist competitive algorithm	0.9783	1
(Markowitz)Traditional return-risk approach	0.2834	2

Table 9 summarizes the ranking of portfolios selected by the imperialist competitive algorithm and the traditional Markowitz approach according to risk and return based on the Sharp or RVAR scale. The results were merely comparable in terms of convergence speed in optimization, so that this comparison can be performed considering number of generation index to reach the optimal solution.

5. Conclusions and recommendations

The present study aimed to optimize multi-objective portfolios using the imperialist competitive algorithm for the top 30 companies listed in Tehran Stock Exchange (TSE). Portfolio selection theory was proposed by Markowitz in 1952. Markowitz developed this theory based on optimization of risk and return of portfolios consisting of several financial assets. The main task of the portfolio selection model was to allocate cash between different portfolios in such a way that risk and return of the portfolio be optimized. Markowitz, in his portfolio selection theory, assumes that all investors make their choices based on two criteria of risk and return. However, many studies criticized the ignorance of other investor preferences in the Markowitz model. Typically, investors consider conflicting preferences and objectives such as return and risk simultaneously when selecting portfolios.
Optimal portfolio selection is one of the important issues in financial literature, which aims to maximize return and minimize the risk of investment considering other preferences. Therefore, one of the main problems of portfolio selection is to select a portfolio of stocks, assets, or securities with conflicting and non-comparable objectives such as return and risk. Due to efficiency of the imperialist competitive algorithm in weighing stock portfolios, it is recommended to combine this technique with fuzzy methods so that performance of this technique be further improved.
Considering that when deciding about formation of a portfolio, the investor must consider several factors (such as risk, return, etc.) and compare these factors with each other when choosing stocks of different companies and invest in the stock which is better than the other available stocks in terms of these factors.

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