



Applying Poisson Equation for Omitting Uneven Contrasts in Brain Tomography Images

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Abstract: *Background and Purpose of the study:* Several important studies have been carried out so far regarding the use of image reconstruction through Poisson equation. During the recent years, after reaching an acceptable level of quality in medical images, the lowering of the calculation complexities was gradually considered to making advancements in the algorithms within a short period of time. Improving the contrast of the medical images was also among the considerations. The present study aimed at applying Poisson equation for the elimination of uneven contrasts in brain tomography images. *Study Method:* CT-scan and MRI images existent in the dataset were utilized as the proposed algorithm's input. These were transformed into gray scale images in the first stage. Poisson algorithm was recalled and used for improving the quality of the images. In the end, the proposed algorithm was compared with two methods, namely back-propagation neural network and genetic algorithm. *Findings:* Considering the results of comparing the proposed algorithm's efficiency with those of genetic algorithm and back-propagation neural network, it can be clearly discerned that the proposed algorithm offered defendable results in contrast to back-propagation neural network plus its considerably faster pace of calculations. However, it was not that much successful in comparison to genetic algorithm. *Conclusion:* The biggest advantage of the proposed algorithm based on screened Poisson equation was its speed of action. The proposed algorithm caused an improvement in the medical images' contrast, especially in the tomographic images taken from such soft tissues as cerebral tissues.

Keywords: *Poisson Equation, Genetic Algorithm and Back-Propagation Neural Network, Brain Tomography Images.*

INTRODUCTION

Pattern recognition methods in medical imaging carry out diagnosis via combining the images obtained from various medical imaging devices and recognition methods. Many of these methods of pattern recognition featuring similarities in terms of their principles and methods are used for assisting the physicians in the diagnosis of such diseases as cerebral infarction, cerebral hemorrhage, breast cancer, gastric cancer and presbyopia caused by diabetes (Kunio, 2007).

During the recent decades, rapid progresses have been made in the creation and blending of various contrast improvement methods, especially adjusted histogram-based methods (Aparma, Anand & Sanjay, 2015; Wang & Zhang, 2013). Histogram integration, as one of the most widely applied methods, has drawn a lot of researchers' attentions for its ease of understanding and application but such problems as the long processing time, calculation complexity in some of the adjusted combined algorithms, loss of fine details, unfavorably

increased contrast and noise amplification, particularly in the investigation of soft tissues like cerebral tissue, have caused tangible declines in the quality of these images during processing (Wong et al., 2016).

In a study that has dealt with proper classifications of the intracerebral hemorrhage, the preliminary classification algorithm was laid on the foundation of a clustering algorithm based on k-means histogram. The results obtained for this algorithm were close to the real information of these patients with cerebral hemorrhage. Moreover, the success of the proposed algorithm in automatic diagnosis of the cerebral hemorrhages was confirmed in brain tomography images (Loncaric et al., 1995).

Cheng et al (1998) introduced another novel technique for automatic classification of brain CT-scan images for isolating the bleeding region. In this method, the attributes of an image were extracted seminally using multi-resolution method. C-means fuzzy clustering was secondly employed to perform the classifications.

In 2005, Mavroforakis et al applied the theory of pattern recognition in medical imaging for the diagnosis of breast cancer. Their work's essence was extraction, selection and analysis of the image data and combining them with the existent clinical properties. They could find a new method of automatic classification using linear classification, neural networks and support vector machine.

Chen et al (2008) demonstrated that the correct breast cancer classification rate could be increased to 94.62% with the selection of the best subset and the most appropriate classification. It is assumed in processing the images obtained from suspicious tumor tissues using evolutionary algorithms that the cancer cells possessed some commonly abnormal states based on which a suitable automatic detector could be designed for the identification of cancer cells.

Yang et al (1997) dealt specifically with CT-scan images' contrast improvement in pulmonary diseases. In this method, wavelet concepts were applied. Rao et al suggested another method for improving the results of image enhancement using image replication technique with sharpness evaluation. Local contrast enhancement increased the sharpness of the dark-light spots. This was more of an optimization problem nature (Rao & Panduranga, 2006).

In another study, the researchers offered a method for improving CT-scan imaging. In this method, gray scale images were used in histogram balance processes. In the meantime, the obtained results were indicative of very good visual improvement of the images (Yin et al., 2006). The other method posited in the study by Wang et al in 2008 was the multiple-level improvement of the medical images. In this method, wavelet transform was utilized in various ways so that the parts of the noises were eliminated and the contrast was increased in every stage.

In a study that was conducted in 2015 on 3D CT-scan images for resolving the background noise problems, static analysis along with computer-based simulation was implemented. The results indicated that the process was promising for the clutter images (Mouton & Breckon, 2015). In 2016, Zhang et al proposed another method of image enhancement. Multi-level image enhancement was achieved using wavelet transform in different ways. In each stage, the noise was eliminated and the image was improved (Zhang et al., 2016).

In a study in 2017, a 3D CT-scan image enhancement algorithm was offered based on order statistic decomposition that was preliminarily used for the removal of the background noises from the CT-scan image segments. Then, the image enhancement algorithm was implemented on these noise-free segments. The algorithm could improve the overall contrast of the image while reducing the noise effects (Peng et al., 2017).

Although the application of screened Poisson equation for image contrast improvement has been frequently taken into account during the recent years, its efficiency has not been so far evaluated in regard of medical images. The present study made use of this algorithm for the first time for minimizing the contrast incongruences existent in images of brain tomography scans. To prove the proposed algorithm's capability, the quantitative information extracted from cerebral tissue images' improvement was evaluated in contrast to the other prominent methods of brain tomography image edition, including genetic algorithm and neural network.

Study Method:

Since the image reconstruction algorithms were based on Poisson equation, many of the applications used this method for compressing the high dynamic domain, contrast improvement and shade removal.

Consider image f . The objective is finding a function u with its vector gradient being close to that of f but with a reduced variance for compensating the brightness incongruence. The objective function serving the variance minimization takes the following form:

$$J(u) = \int_{\Omega} \|\nabla u - \nabla f\|^2 dx + \lambda \int_{\Omega} (u - \bar{u})^2 dx \quad (1)$$

Where, λ is a fixed value controlling the balance between both the existent predicates.

Euler-Lagrange differential equation is nonlinear. The equation becomes linear if assumed that mean u value matches the mean f value. Under such a condition, function u that minimizes functional J holds true in the following Euler-Lagrange equation with Newman’s homogenous boundary condition $\frac{\partial u}{\partial n} = 0$, on $\partial\Omega$:

$$\lambda(u - \bar{f}) - \Delta u + \Delta f = 0, \text{ on } \Omega \quad (2)$$

Where, n is the normal boundary vector. Equation (2) is known as screened Poisson equation. It has been proved that imposing of a mean value onto the solution is insignificant.

Assume the following problem:

$$\begin{cases} \lambda u - \Delta u = -\Delta f + \lambda k & ; \text{in } \Omega \\ \frac{\partial u}{\partial n} = 0 & ; \text{in } \partial\Omega \end{cases} \quad (3)$$

Where, k is a constant and $\lambda > 0$. It is evident that if $\lambda \neq 0$, problem (3) would only have a single solution. It is assumed that u_1 and u_2 are two solutions for problem (3) with k_1 and k_2 constants, respectively. Then, considering the unique solution to problem (3), the following is mentioned:

$$u_1 - u_2 = k_1 - k_2 \quad (4)$$

In working with 8-bit digital images, the image values range from 0 to 255. When solving problem (3) for the images, the final solution should give results within the aforesaid ranges. The method allows the achievement of relative contrast advantages.

Fourier Transform is defined as follows for $f(x,y)$:

$$\hat{f}(w_x, w_y) = \int_{\mathbb{R}^2} f(x,y) e^{-ixw_x} e^{-iyw_y} dx dy \quad (5)$$

And, inverse Fourier transform takes the following form:

$$f(x,y) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} \hat{f}(w_x, w_y) e^{ixw_x} e^{iyw_y} dw_x dw_y \quad (6)$$

It has to be reminded that,

$$\hat{f}_x = iw_x \hat{f}, \hat{f}_y = iw_y \hat{f}, \hat{f}_{xx} = -w_x^2 \hat{f}, \hat{f}_{yy} = -w_y^2 \hat{f} \quad (7)$$

Then, screened Poisson equation’s Fourier transform gives:

$$\lambda \hat{u} + (w_x^2 + w_y^2) \hat{u} = (w_x^2 + w_y^2) \hat{f} \quad (8)$$

And, solving it for \hat{u} gives the following formula:

$$\hat{u} = \frac{w_x^2 + w_y^2}{\lambda + w_x^2 + w_y^2} \hat{f} \quad (9)$$

The algorithm proposed based on screened Poisson equation has been displayed in the following equation:

Data: An input image (one channel) f , $\lambda > 0$ tradeoff parameter and s the percentage of saturation of the simplest color balance.

Result: The image solution of screened Poisson equation u .

```

begin
    Simplest color balance of  $f$  with  $s\%$  of saturation;
    FFT of  $f$ 
    Solve equation using (23-3)  $\rightarrow \hat{u}$ ;
    IFFT of  $\hat{u} \rightarrow u$ ;
    Simplest color balance of  $u$  with  $s\%$  of saturation;
end
    
```

The algorithm proposed in the present study has been simulated using MATLAB software and the CT-Scan and MRI brain images have been applied as the datasets. Each image existent in the dataset has been inserted in the proposed algorithm as the throughput and it would be converted to gray scale in the first stage. Poisson algorithm was recalled and used for image quality improvement. During running Poisson algorithm, Laplace formula was firstly implemented on the input data or the very normal images. In this state, each pixel was considered with four neighbors, upper, lower, left and right neighbors, and this action was also undertaken for all of the aforementioned neighbors. Afterwards, the differentials were calculated and the specific values were calculated following the exertion of Fourier Transform. The results were shown following the exertion of inverse Fourier transform. In quantitative evaluation of the proposed algorithm, back-propagation neural network was used for its possession of self-correction and reciprocity. It caused the improvement as well as genetic algorithm that could successfully work for image optimization in an acceptable speed (Hole, Gulhane & Shellokar, 2013).

Findings:

Figure (1) illustrates one image existing in the dataset so the proposed algorithm could be used for improving its quality.

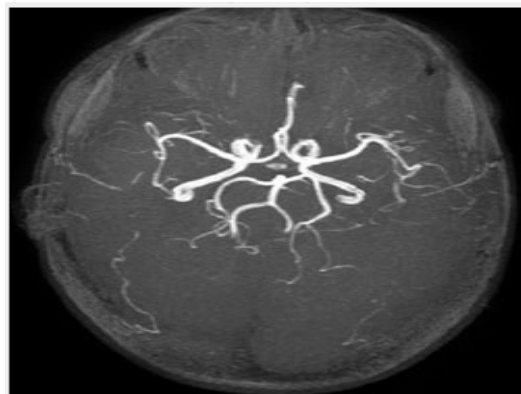


Figure 1: Another example of the images existent in the dataset

The quality improvement of the image shown in figure (1) began with the implementation of the proposed algorithm. The preliminary results related to Poisson relation have been displayed in figure (2). The examination of this figure indicated that the input image had undergone changes parallel to the quality

improvement. It is evident that the more the number and volume of these changes, the higher the image quality would be so the optimal output would be increased.

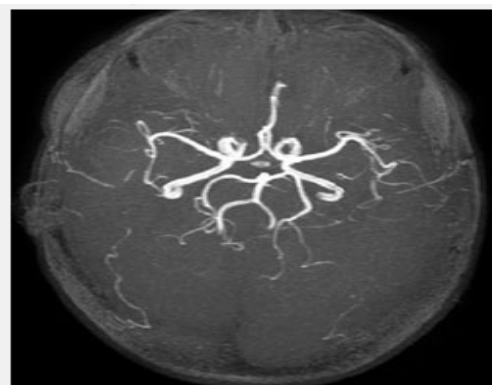


Figure 2: Improvement process using Poisson relationship

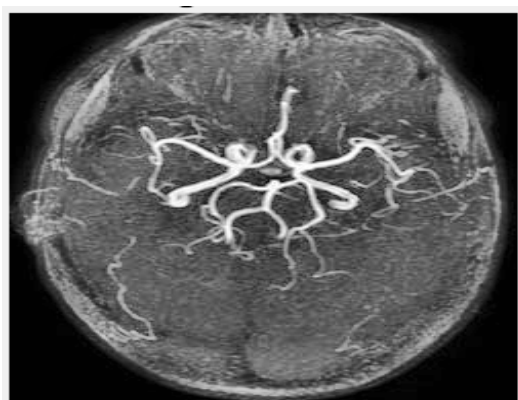


Figure 3: The second improved image using the proposed algorithm

The final improved image has been demonstrated in figure (3) and figure (4) which depicted the initial image and the improved image at the side of one another so that they could be readily compared. The histograms pertaining to the images shown in figure (4) have been portrayed in figure (5). The investigation of figure (5), as well, was suggestive of the idea that the improvement process had caused the image histogram to become extended and also the various components existent in the image to be shown with a higher precision in the image.

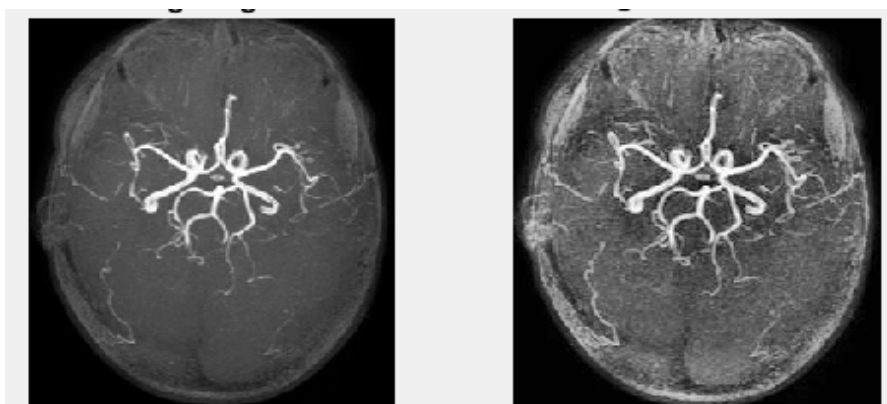


Figure 4: The main image existent in the dataset (left) and its corresponding improved image (right)

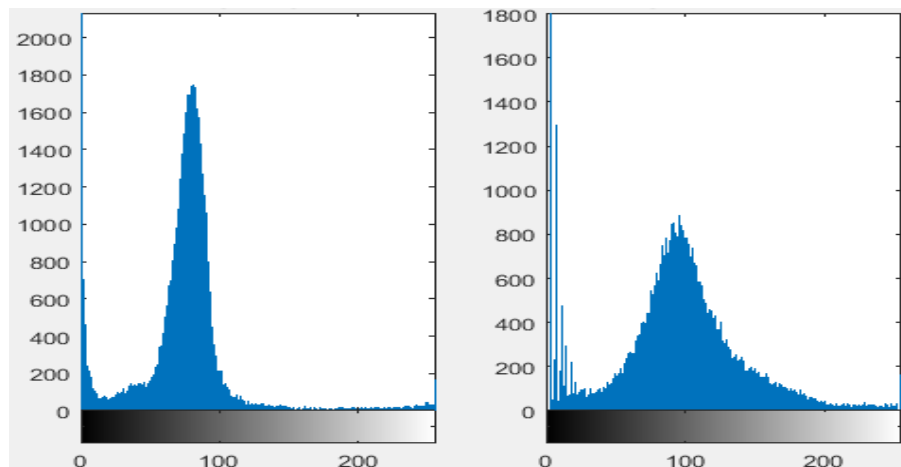


Figure 5: Histograms of the main image (left) and the improved image (right)

On the other hand, the results obtained from the comparisons using SSIM instruction existent in MATLAB software have been illustrated in figure (6). The run time and PSNR of the output could be calculated and shown for the second input image existent in the dataset, as well. Figure (7) displays the run time and PSNR pertinent to the second input image.

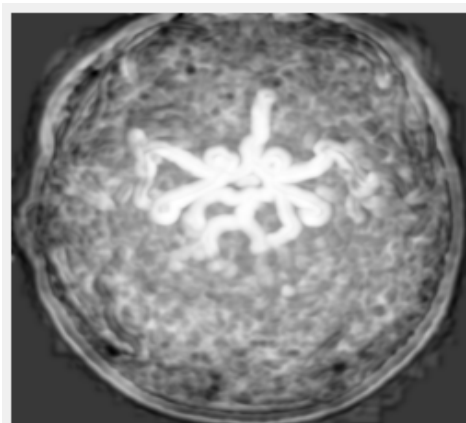


Figure 6: Results obtained from SSIM instruction for the second input image

```
Command Window
Solving Poisson Equation Using DST
Time for Poisson = 0.14558 second
SSIM Values = 46.2263

The Peak-SNR value is 17.9552
The SNR value is 7.6545
Elapsed time is 2.779291 seconds.
fx >> |
```

Figure 7: PSNR and run time pertaining to the second input image

Up to here, the changes in the histograms of the images related to the proposed algorithm have been demonstrated. In the forthcoming section, the evaluation criteria of the proposed algorithm and the other two methods, namely neural network and genetic algorithm, would be examined.

Table 1: Investigating the evaluation criteria for the proposed algorithm and some of the other existing methods

Method used	Run time	PSNR	SSIM
Proposed algorithm	3.82637	17.9652	46.6322
Neural network	17.43673	12.9573	39.572
Genetic algorithm	12.67394	19.0838	48.7364

Investigation of table (1) showed that the algorithm proposed in the current research paper took a shorter period of time for image quality improvement as compared to the other existing methods. On the other hand, the proposed algorithm could offer higher PSNR and SSIM values in contrast to the methods using back-propagation neural network for the image quality improvement. However, the comparison of the proposed algorithm with the method using genetic algorithm for image quality improvement was reflective of lower PSNR and SSIM values for the former. The biggest advantage of the algorithm proposed herein based on screened Poisson equation was its speed of action.

Discussion and Conclusion:

At present, processing techniques play a considerable role in medicine profession by decreasing the need for human workforce as well as overcoming the problems resulting from human error and the reduction of the treatment costs to some extent. X-ray images, especially brain images include different regions with various details. Therefore, sharp and soft transfers between the regions and details exist in visual coverage. When all the details are increased by the same size, the relatively lower details are prevented from being detected in practice. On the other hand, as the brain tomography images are used for diagnosis purposes, image improvement should not cause the creation of misleading information. In brain tomography images, sharpness gradient unevenness comes about due to the radio frequencies' discoordination or static magnetic field changes. Thus, the image contrast should be adjusted before its reconstruction.

In simple terms, the contrast function of an image's pixels is considered as a mathematically discrete function hence no derivative is specified for it. The solution lies in the creation of a continuous function for these points that takes an identical value for each spot. On the other hand, there is no function the vector gradient of which completely matches the gradients of the input image. Here, Poisson equation introduced a function the gradient of which was closest to the gradient of the input image but with reduced variance so that the image's sharpness incongruence could be made up.

Application of Poisson equation contributed to the elimination of uneven sharpness and contrast in brain tomography images and, resultantly, the dark regions of the images could be better observed and the image details could be preserved more uniformly.

Next, for reducing the objective function's variance, the need for minimizing Poisson equation became well evident. To do so, Euler-Lagrange equation was applied. The equation was shown in the form of screened Poisson equation.

To prove the independence of objective function's mean and image contrast, it was necessary for image contrast improvement and signal processing feasibility to take it from time space to frequency space. This stage was carried out using fast Fourier transform.

The image was symmetrically and alternatively stretched to the edges and became several times more magnified with the exertion of such a transform under Newman's boundary condition. Then Lambda constant was increased with image's entry into frequency domain for the reduction in sharpness non-uniformity and

the frequency filter acted as a high-pass filter. Doing so made the image optimized in frequency area or, in other words, the contrast of the image was enhanced.

In the next stage, to be able to observe the image, it was necessary to return it from frequency domain to time domain which was done using inverse Fourier transform.

Next, the histograms of various brain tomography images, incorporating the main image and the improved image based on the proposed method, were delineated. As it was shown, the proposed method caused the image histogram to be more expanded and the various parts of the image could be seen with a higher precision and clarity. The combination of various methods was advised for the proposing of novel algorithms capable of finishing image improvement process within a shorter period of time with higher SSIM and PSNR.

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