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# Noise Spectral Estimation Methods and Implementation of an Algorithm in the Frequency Domain for Improving Detection in a Passive Sonar System

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Abstract: The sonar is actually a radar which is used on or under the surface of the water and uses sound waves instead of radio waves to survey the environment. Detection and classification of marine vessels based on their acoustic radiated noise is of great importance in sonar systems. The non-static nature of the radiated noise of marine vessels and its dependency on the propagation channel, the variability of some of its parameters, and the difficulties in the simulation of this noise, make detection algorithms using a limited number of real data with a low diversity and or simulated training data to be not adequately valid. The present study aims to provide a new algorithm for the separation of target signals received in the form of convolutive mixture. For this purpose, a time-frequency window is designed that reconstructs an approximation of the target signal in the frequency domain. The cosine of the angle between two vectors is equal to the cosine of a parameter, called Hermitian angle between the two vectors. The Hermitian angle between two complex vectors remains unchanged if the two vectors are multiplied in a complex scalar. In the present study, this property is used to design a window for the separation of sources mixed convolutively. The advantage of this window over other detection methods is that there is no need for primary information on the geometric location of sources and hydrophones.

**Keywords:** Inactive sonar, Frequency-time window, Hermitian angle.

#### INTRODUCTION

Ships, vessels and submarines need sonar systems to communicate and identify their surroundings. In fact, the sonar acts as the submarine or ship's eyes. Today's modern sonar are relatively new equipment and their history backs to World War II and onward. Generally, target detection algorithms are used in three time, frequency, and time-frequency domains. Urick (1983) conducted a combined experimental-numerical study to study Sonar Equation for calculating the signal-to-noise ratio in the time domain. Nielsen (1991) reported that the DEMON analysis provides the properties of a propeller, such as the number of shafts, the shaft rotational frequency, and the target speed rate. Dawe (1997) reported that using Receiving Operating Characteristics (ROC) curve improves the detection performance in Sonar Equation. Chin et al. (2000) used Two-Pass Split-Windows (TPSW) algorithm and the neural network to classify under water signal. Abraham (2010) used non-Gaussian functions to determine the Detection Threshold (DT). Wakayama et al. (2011)

described the anticipating of probability of target presence in an area. Kil Woo et al. described the DEMON algorithm for detecting a target in a passive sonar (Chung et al., 2011). The method reported in (Diamant, 2016) focused on the Normalized Match Function (NMF). The NMF is used when the noise covariance matrix is rapidly changing and is hard to estimate. In the frequency domain, Martino carried out Low Frequency Analysis and Recording (LOFAR) broadband analysis, which estimates the noise fluctuation of the target machinery (Di Martino, Haton and Laporte, 1993). Borowski et al. (2008) examined underwater signal analysis in frequency domain. Zhishan Zhao et al. (2017) proposed an improved matched filter in additive white Gaussian noise (AWGN) combined with the adaptive line enhancer by analyzing the output spectrum and it should be noted that this method was proposed for active sonar. In the time-frequency domain, Mora et al. performed an independent component analysis for the detection and classification of signals against background noise (Zhan et al., 2016).

In the present study, considering the frequency of data used, a troop model is used for modeling the sound absorption in water. In the second section, considering the available data, the method proposed for detecting targets in passive sonar, is examined and simulated. In the third section, the effect of Gaussian noise and source position on the detection algorithm is investigated. Finally, in Section 4, a conclusion and several suggestions for future studies in this area are presented.

### Proposed Method

In the proposed method, an algorithm is presented for the design of a window for the separation of convolutive mixtures. One of the advantages of the proposed algorithm is that there is no need for any geometric information about the location of hydrophones and targets.

Another advantage of this method is the direct use of known clustering algorithms used for window estimation and also there is no significant increase in the computational burden of window estimation as the number of hydrophones increases.

The mathematical expression of the convolutive mixtures of signals is as follows:

$$x_{p}(n) = \sum_{q=1}^{Q} \sum_{l=0}^{L-1} h_{pq}(l) s_{q}(n-l)$$

$$p = 1, ..., P$$

$$q = 1, ..., Q$$
(1)

Where, P denotes the number of observations, Q the number of sources, and L the length of mixing filter.  $X = \begin{bmatrix} x_1, x_2, ..., x_p \end{bmatrix}^T$  is the vector of our observations,  $x_p(n) = \begin{bmatrix} x_p(0), ..., x_p(N-1) \end{bmatrix}^T$  is also a set of mixture samples in the output of the pth hydrophone, where N denotes the number of samples. The impulse response of the qth source to the pth hydrophone is  $h_{pq}(l), l = 0, ..., L-1$ . In general, in those cases where observations are assumed to be convolutive, frequency- or time-frequency-based methods are more appropriate than a time-based method, because the signal combination model changes from convolutive to linear in a time-based method is used.

Eq. (1) in the time-frequency domain is given by Eq.(2):

$$X(k,t) = \overline{H}(k)S(k,t) = \sum_{q=1}^{Q} H_{q}(k)s_{q}(k,t)$$
(2)

 $X(k,t) = [X_1(k,t),...,X_p(k,t)]^T$  is a column vector of the time-short Fourier transform coefficients of the output of the hydrophones in the k<sup>th</sup> frequency bin in the time frame, t. The column vector of STFT

coefficients of sources is  $S(k,t) = [S_1(k,t),...,S_Q(k,t)]^T$ .  $H_q(k) = [H_{1q}(k),...,H_{pq}(k)]^T$  is the qth column vector of the mixing matrix in the kth frequency bin.

 $\overline{H}(k) = [H_1(k), ..., H_Q(k)]$  is the mixing matrix in the  $k^{th}$  frequency bin, and  $H_{pq}(k)$  is the  $k^{th}$  DFT coefficient of the impulse response of the  $q^{th}$  source to the  $p^{th}$  hydrophone, and, the impulse response is assumed identical at all times.

The use of data simulations has significant advantages and in a case where real data are not available or the amount of available data is limited, in order to examine, test and modify sonar algorithms, data simulation should be inevitably used (Karimi, 2003). Figure (1) shows the block diagram of the proposed method.

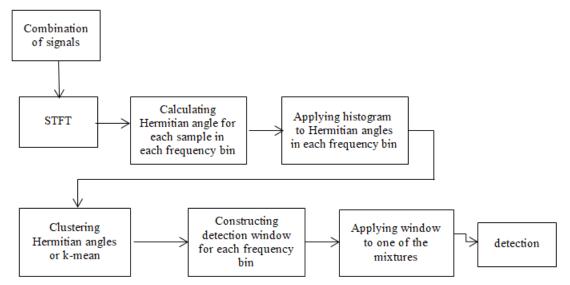


Figure 1: Block diagram of the proposed method

As shown in Fig. (1), the proposed method is implemented through six blocks. First, the received signal is transmitted to the time-frequency domain, and then a parameter called Hermitian angle is defined. Using the property of this parameter and clustering methods, the separation window is designed. To simplify the proposed method, the block diagram shown in Fig.(1) is described in three phases.

### Phase I: A mixer with real values

First, the mixer is considered with real values, then, the impulse response will be a scalar vector with the domain  $h_{pq}$  (the element (p, q) of the mixing matrix). Since the impulse response is a scalar vector,  $Im\{H_{pq}(k)\}=0$ ,  $Re\{H_{pq}(k)\}=h_{pq}, \forall k$ ,  $H_{q}(k)=h_{q}=[h_{1q},...,h_{pq}], \forall k$ , where  $h_{q}$  is the  $q^{th}$  column of the mixing matrix in time domain, and  $H_{q}(k)$  is the  $q^{th}$  column of the mixing matrix in frequency domain in the  $k^{th}$  frequency bin. To simplify the problem, P=Q=2. It is assumed that there are only the source components at the point  $(k_1, t_1)$ .

$$X(k_1, t_1) = H_1(k_1)S_1(k_1, t_1)$$
(3)

The above equation can be rewritten as follows:

$$\operatorname{Re}\{X(k_1,t_1)\} + j \operatorname{Im}\{X(k_1,t_1)\} = H_1(k_1) \left(\operatorname{Re}\{S_1(k_1,t_1)\} + j \operatorname{Im}\{S_1(k_1,t_1)\}\right)$$
(4)

Re $\{S_1(k_1,t_1)\}$  and Im $\{S_1(k_1,t_1)\}$  are real. Therefore, by comparing the real and imaginary parts of Eq.(4), it is observed that the direction of the column vectors Re $\{X(k_1,t_1)\}$  and Im $\{X(k_1,t_1)\}$  is the same as that of  $H_1(k_1)$  and the first column of the mixing matrix. At another moment, such as $(k_2,t_2)$ , with the assumption that there is only the source  $S_2$ , like the moment  $(k_1,t_1)$ , the direction of the column vectors Re $\{X(k_2,t_2)\}$  and Im $\{X(k_2,t_2)\}$  is the same as that of  $H_2(k_2)$  and the second column of the mixing matrix. By assuming the sparsity of sources in the time-frequency domain, the sparsity scheme of Re $\{X(k,t)\}$  and Im $\{X(k,t)\}$  shows the clear orientation towards the known directions of the column vectors of the mixing matrix, and hence one can determine the mixing matrix and estimate sources.

#### Phase II: A mixer with complex values

When the mixer is considered convolutive, column vectors  $H_q(k)$  are complex in Eq.(1). By multiplying these complex vectors by the complex scalar  $S_q$  (k, t), the angle of the vectors with complex values varies. Consider two complex vectors. The cosine of the angle between these two vectors is defined as:

$$\cos(\theta_c) = \frac{u_1^H u_2}{\|u_1\| \|u_2\|} \tag{5}$$

Where,  $||u|| = \sqrt{u^H u} \cdot \cos(\theta_c)$  can be expressed as Eq.(6):

$$\cos(\theta_c) = \rho e^{j\varphi}, \rho \le 1 \tag{6}$$

$$\rho = \cos(\theta_H) = \left| \cos(\theta_c) \right| \to \theta_H = \arccos(\rho) \tag{7}$$

Where  $0 \le \theta_H \le \pi/2$  and  $-\pi \le \varphi \le \pi$ . The angle between the two vectors  $u_1$  and  $u_2$  is called Hermitian angle (Scharnhorst, 2001).

The Hermitian angle between the two vectors remains unchanged if the two vectors are multiplied by a complex scalar. In the present study, this property is used to design a blind source separation window and our observations are convolutive. The multiplication of a complex vector by a complex scalar does not affect the Hermitian angle between this vector and the other vector (reference), so it is considered the reference vector contains P elements with 1+1j values. The Hermitian angle between the reference vector r and  $H_q(k)$  is always constant, even if  $H_q(k)$  is multiplied by the complex scalar  $S_q(k,t)$ . Assuming that sources are sparse in the time-frequency domain, each point of the time-frequency plane contains only one of the source components, and the Hermitian angle between the reference vector and the observations  $\mathbf{X}(k,t)$  at that point is equal to the Hermitian angle between the reference vector and  $H_q(k)$  corresponding to  $S_q(k,t)$ . Hence, samples of the observations in each frequency bin are classified in Q classes with a clear orientation, according to the reference vector, and all samples of a class belong to a source.

#### Phase III: Construction of window and detection using the proposed method for P = Q = 2

In general, when there are P observations and Q sources, in order to calculate the Hermitian angle at  $t_1$ , the Hermitian angle between the reference vector and the observation vectors  $X(k_1, t)$ ,  $\forall t$  is calculated as follows (Scharnhorst, 2001):

Specialty J. Eng. Appl. Sci., 2019, Vol. 4 (2): 114-128

$$\Theta_H^{(k_1)}(t_1) = \cos^{-1}\left(\left|\cos\left(\theta_C(k_1, t_1)\right)\right|\right) \tag{8}$$

$$\cos(\theta_{C}(k_{1},t_{1})) = \frac{X(k_{1},t_{1})^{H} r}{\|X(k_{1},t_{1})\|\|r\|}$$
(9)

The vector of the Hermitian angle calculated in the  $k^{th}$  frequency bin is used for classification of samples of observations and construction of the window  $M_q(k,t)$ . By multiplying the constructed window by the observations, an estimation of the separated signals is obtained.

$$Y_q(k,t) = M_q(k,t)X_p(k,t), \forall t,q = 1,...,Q$$
 (10)

In reality, the full sparsity of sources is impossible. Therefore, it is necessary to classify the Hermitian angles obtained in each frequency bin to a number of classes corresponding to the number of targets. Clustering algorithms can be used to classify values  $\Theta_H^{(k)}$  and their corresponding samples of observations. One of the most widely used clustering algorithms is kmeans, which is used here to classify the samples of observations. Since the kmeans algorithm is a hard classification method, each sample belongs to one of the clusters and the membership function is binary (0 or 1). We know that the starting center of the clustering algorithm affects the final center of the clusters. To select the initial starting center of the algorithm, the histogram method is applied to data  $\Theta_H^{(k)}$ . The middle centers with the highest frequencies in the histogram are used as the initial centers of the kmeans algorithm. The result of the clustering algorithm, which is in square pulse, is applied as a window to one of the observations to reconstruct an approximation of one of the signals. It is adequate to apply the inversion of the window to the same observation to reconstruct another signal.

A set of acoustic data of a passive SONAR system, including the sounds of the propeller, ships, oil tankers, etc. from the Mendeley data website. The underwater environment with a depth of 200 m and the sound velocity of 1500 m/s is considered. The widths of the signals are made identical. Considering two sonar targets on the simulated environment surface, the convolutive mixture of signals is obtained from two bottom-mounted hydrophones that are spaced 10 meters from each other, at a distance of 2 km from the coast and a depth of 200 m.

#### Results

## 1. Results of simulation of the proposed method for P = Q = 2

The simulated scenario is shown in Fig. 2, in this figure, the overall channel response is drawn from each source to each target and with the reflection path.

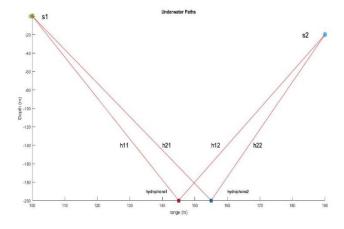


Figure 2: The simulated scenario

Two acoustic data related to the boat and container are considered as two target signals. These two signals are shown in Fig. 3:

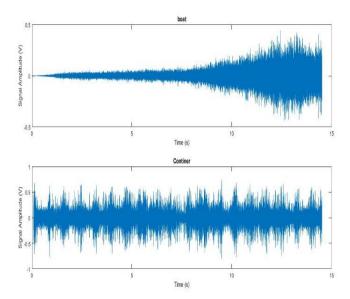


Figure 3: Initial signals (Karimi, 2003)

After the signals pass through the paths, a mixture of signals is received by hydrophones. An example of impulse responses of paths is shown in Fig.4:

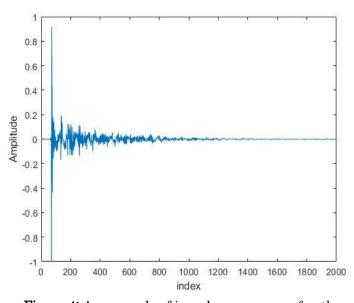


Figure 4: An example of impulse responses of paths

In Fig. 5, the output of the hydrophones is shown:

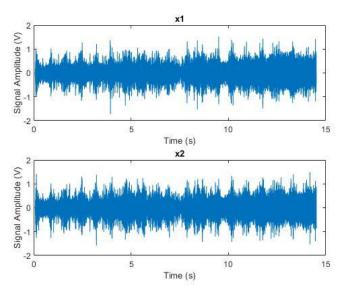


Figure 5: Output of the hydrophones

First, by applying the hanning window, shown in Fig. 6, to the observations and then applying the Fourier transform to each bin, the signal is transmitted to the frequency domain.

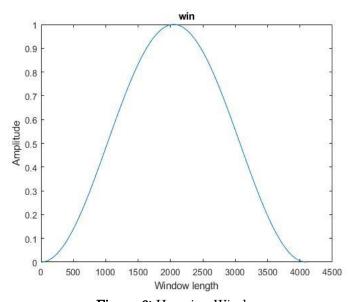


Figure 6: Hanning Window

Then, the Hermitian angle between the reference vector and the observation vectors, X(k,t),  $\forall t$  in each frequency bin is calculated. The calculated angles for the samples in the first to third bins will be in the form shown in Fig.(7):

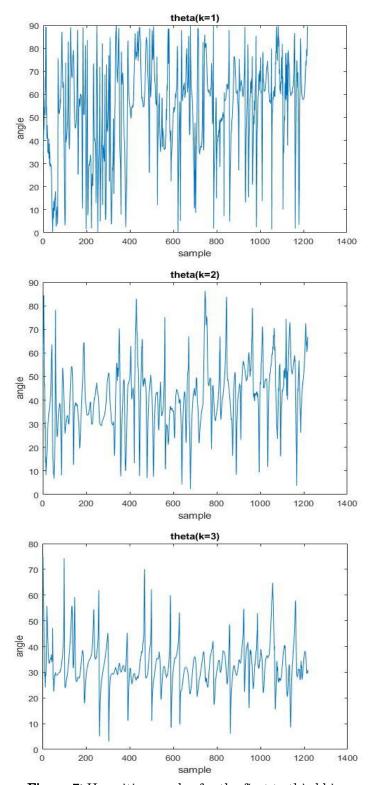


Figure 7: Hermitian angles for the first to third bins

To cluster the Hermitian angles between the reference vector and the observation vectors, the histogram of the Hermitian angles are plotted and the centers of two angles with the highest frequency in the histogram are considered as the initial centers of the kmeans clustering algorithm. For example, according to the histogram shown in Fig. 8, in the third frequency bin, the centers of 35.46 and 28.27 are the most frequent.

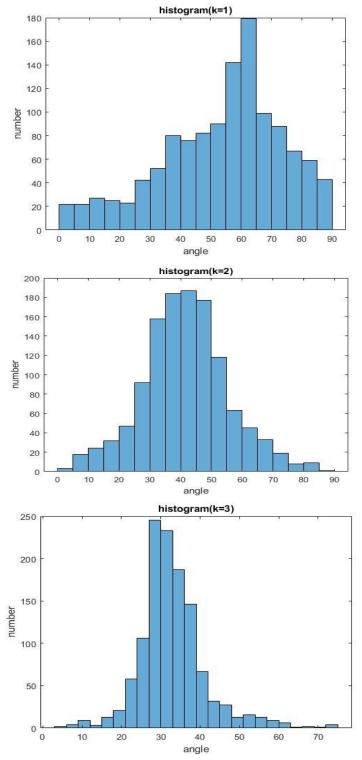


Figure 8: Histogram of Hermitian angles for the first to third bins

Considering the two values obtained from the histogram as the initial centers, the kmeans algorithm classifies the Hermitian angles between the reference vector and the observation vectors in two binary clusters. The result of the kmeans algorithm is considered as a separation window.

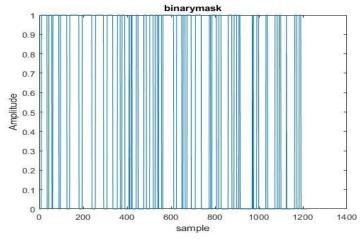


Figure 9: The window produced by the kmeas algorithm for the frequency bin k = 3

Finally, by applying the separation windows of the frequency bins and its inversion to one of the observation vectors, one can separate the source signals. By putting the separated parts of the signals together, an estimation of the target signals is reconstructed.

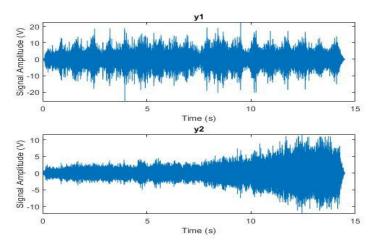


Figure 10: Reconstructed target signals

#### 2. Investigation of the proposed algorithm with different sonar targets

To investigate the detection accuracy, several sets of acoustic data were applied to the proposed algorithm. Using the Neyman-Pearson correlation criterion, the similarity between the separated acoustic data and the initial acoustic data is calculated. In Table (1), the results of detection are shown for a case where the hydrophones are spaced 10 meters from each other.

Table 1: The degree of similarity between the initial data and the detected data

	Initial data	Pearson's correlation
First group of targets	Oil tanker	93%
	Motor boats	67%
Second group of targets	Ship	84%
	Pusher	75%

Supertanker	89%	
Launch	80%	

## 3. Investigation of the effects of noise in the proposed method

For this purpose, the signal to Gaussian noise ratio is increased in several steps, and the corresponding detection of each change is calculated. The results are shown in Table (2).

Table 2. Accuracy of detection per SNR changes			
SNR	Accuracy of boat detection	SNR	Accuracy of oil tanker detection
20	21	20	65
40	35.5	40	72
60	42.4	60	85
80	65.5	80	93

Table 2: Accuracy of detection per SNR changes

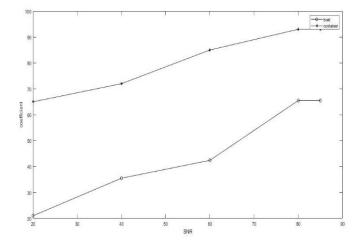


Figure 11: The effect of Gaussian noise on the detection accuracy of the proposed method

One of the noises affecting the background noise in the underwater environment is wind noise. This noise is in the frequency range from 100 to 1000 Hz. To simulate this noise, the ARMA statistical model with a degree (1 and 2) was used. This noise is in the form of Eq.(9) (Karimi, 2003).

$$y(n) = 0.76y(n-1)-0.3496y(n-2)+x(n)-0.7x(n-1)$$
 (9)

This noise is added to the passive sonar signal in two ways: in an additive form and with mixpad<sup>1</sup>. The accuracy of oil tanker detection decreases by 3% in the case of adding noise in an additive form and by 10% in the case of adding noise with mixpad.

Classification accuracy (%)

Original signal Original signal+wind noise Mixture of original signal and wind signal with Mixpad

Oil tanker 89.38 82.24

Table 3: Effect of wind noise on the accuracy of the proposed method

<sup>1 &</sup>quot;Go Mix It!" [Online]. Available: http://gomix.it/. [Accessed: 15- Dec-2014]

Motor boat 64.89 55.47	
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### 4. The use of cmeans instead of kmeans in the proposed method

The cmeans algorithm clusters the samples respecting the membership values that have an inverse relationship with distance. For example, if it is estimated that the distances between a sample and cluster centers are equal, the kmeans algorithm allocates the sample to a cluster, while in the same conditions, the cmeans algorithm allocates the sample to all clusters with the same membership values. Therefore, the window estimated with the cmeans algorithm is smoother, improving the performance of the detection method. Fig.12 shows the performance of the suggested detection method in the case of container detection in the first target group using two clustering methods.

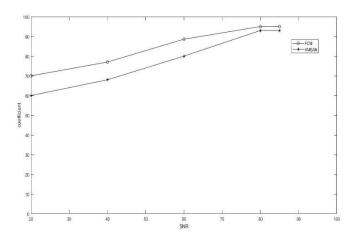


Figure 12: The performance of proposed detection method in the case of container detection with two clustering methods

As shown in Fig. 12, the proposed detection method with cmeans clustering algorithm has a better performance at different signal-to-noise ratios.

#### 5. Specific target detection

Considering the acoustic data used in the present study, a training set is developed for a classifier algorithm. In the training set, a signal is considered as a target and the rest of the signals as non-target. The label 1 is considered for the target signal and label 0 for non-target signal. After applying the proposed method to observations and obtaining separated signals, all the separated signals are applied to the knn algorithm, if the target is in the separated signals and as a result, the target signal is identified by label 1.

#### 6. Comparison of detection algorithms

First group of

targets

Oil tanker

Motor boat

In the present study, a new algorithm is proposed for target detection in the time-frequency domain. The results of this method are compared with the three general passive sonar detection methods: 1- using sonar equation; 2- based on DEMON and Independent Component Analysis (ICA); and,3- using adaptive threshold. To compare the proposed method with the three aforementioned methods, a database was used. In the present study, two sets of test data were considered. The former set includes boats and oil tankers, the latter includes mid-sized ships and pushers. In Table 4, the results of the comparison are listed.

Initial data

Accuracy detection

Proposed method | Sonar equation | DEMON | Adaptive threshold

**Table 4:** Comparison of detection methods

Second group of	Ship	79%	460/	69%	OE0/
targets	Pusher	1970	46%	09%	85%

As shown in Table (4), for the first target group, the actual detection rate of passive sonar target detection using adaptive threshold, DEMON and the Sonar equation are 81%, 51% and 30%, respectively, while the actual detection rate of the proposed method is 80%. The results show that the correct detection rate of the proposed method is improved compared to the DEMON method and is approximately equal to the adaptive threshold.

## 7. Performance of proposed method with increasing the number of targets

Fig. 13 shows the Hermitian angles in the third frequency bin in the case of the presence of three targets of oil tanker, ship and boat.

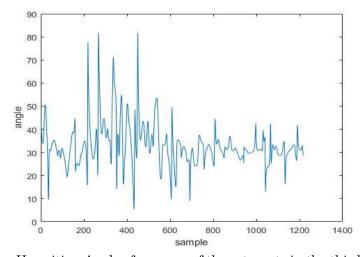


Figure 13: The Hermitian Angles for a case of three targets in the third frequency bin

Assuming the presence of three targets, the proposed method is used to detect all three sources. The results of the detection are listed in Table (5).

The results of the suggested method in a case of three targets			
	Initial data	Pearson's correlation	
	Oil tanker	90%	
First group of targets	Motor boat	67%	
	Ship	79%	
	Ship	84%	
Second group of targets	Pusher	75%	
	Launch	70%	

**Table 5:** The results of the suggested method in a case of three targets

## Conclusion

In the present study, a method was proposed for the separation of target signals, which are in the form of convolutive mixture, in the time-frequency domain. The proposal of this study was an algorithm for estimating the time-frequency window to be applied to observations to separate targets in the frequency domain. The window estimation algorithm is based on the concept of angle in a complex vector space. The advantage of this window over other detection methods is that there is no need for basic information on the geometric location of sources and hydrophones. Using the kmeans clustering algorithm, the mixture samples are classified according to the Hermitian angle, and the result of the kmeans algorithm is directly used as the separation window. The kmeans clustering algorithm is a hard classification method. That is why, sudden

changes take place in the amplitude of the reconstructed signal. This sudden changes in the amplitude result in the unrealistic display of the signal at some times. The cmeans algorithm was used to solve this problem and improve the performance of detector. The results of this method was compared with the three general passive sonar detection methods: 1- using sonar equation; 2- based on DEMON and Independent Component Analysis (ICA); and,3- using adaptive threshold, and the results showed the improved performance of this method compared to the two methods of sonar equation and Independent Component Analysis (ICA).

The advantage of the proposed method to the previous methods is that, unlike previous methods, it does not require any estimate of the mixing matrix or any information on the location of sources for the window estimation. Moreover, it is not susceptible to weather, night and day, search environment type and it is simple to implement and use.

Unfortunately, due to limitations in the sonar data collection, it was not possible to test the presented methods on other surface and undersurface vessels. In any case, the following suggestions are presented for future studies:

- 1- Due to the fact that after the detection using the proposed method, there is scale ambiguity and permutation ambiguity in separated signals, it is necessary to provide methods for eliminating these two ambiguities.
- 2- Considering the sparsity of natural acoustic signals in the frequency domain, it is suggested to implement some algorithms in the time domain for detection and compare the results with the results obtained in the frequency domain.
- 3- Given the fact that the Doppler effect and signal degradation due to the extent of the environment are ignored in this study, it is suggested to investigate the proposed method considering these two effects.
- 4- In this study, it was assumed that the total number of sources was known. In real world, the number of target sources is unknown. Therefore, it is suggested to propose a method for estimating the number of sources before classifying the Hermitian angles with a clustering algorithm.
- 5- In this research, simulation was performed with the assumption that the channel remains unchanged over time. It is suggested to assume that the channel is changeable over time.

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