Science Arena Publications
Specialty Journal of Electronic and Computer Sciences
ISSN: 2412-7485
Available online at www.sciarena.com
2019, Vol, 5 (3): 43-53

Calibration of Microelectromechanical Accelerometer Used in Measurement While Drilling Tools

Mohammad Hasan Moradi [1] , Mehdi Izadi [2*]

[1]Professor of Electrical Engineering, Department of Electrical Engineering, Faculty of Engineering, Bu Ali Sina University, Hamedan, Iran.

[2]Master of Electrical Engineering, Department of Electrical Engineering, Faculty of Engineering, Bu Ali Sina University, Hamedan, Iran

***Corresponding Author**

Abstract: *Today, directional drilling of oil wells serves to increase the exploitation of underground oil reserves. Moreover, acquiring the exact knowledge of the spatial position by measuring the inclination and azimuth and finding the three-dimensional well coordinates for controlling the drilling path is a necessity actualized using the measurement while-drilling (MWD) tools including accelerometers and magnetometers, which are manufactured by the microelectromechanical systems (MEMS) technology. The microelectromechanical systems technology is a combination of microelectronics (electronic integrated circuits), micro-machining and complex mechanical systems. These sensors reduce energy consumption, costs, volume, and weight and improve reliability and speed. However, their measurement is erroneous and lacks adequate accuracy. Hence, the calibration of these sensors in addition to the calibration by the manufacturer is necessary for reducing these errors and improving the measurement accuracy. In this paper, 2 methods of calibrating the microelectromechanical accelerometer sensors are proposed. To this end, a mathematical model is developed for the accelerometer and the model parameters (the scale factor, bias, and misalignment) are determined using the least squares error method and the particle swarm optimization algorithm. The compliance between the output components of the accelerometer sensor and the actual acceleration components is a criterion for assessing the model parameters. According to the results, these methods yield models with fewer errors and improve the performance of the acceleration sensors.*

Keywords: *Calibration, Microelectromechanical Systems, Accelerometer and Magnetometer, Measurement While-Drilling, Least Squares Error, Particle Swarm Optimization Algorithm.*

INTRODUCTION

Directional drilling is the science of directing a well along a predesigned path towards a subsurface target whose horizontal displacement and orientation in the perpendicular direction are predetermined. The most important advantage of directional drilling controlled with the measurement tools is its cost-effectiveness and increased exploitation of the underground oil and gas reserves. Today, the measurement while-drilling tools including tri-axis accelerometers and tri-axis magnetometers of microelectromechanical systems (MEMS) are used to estimate position in the directional drilling of oil wells which are combine of microelectronics and micro machining. Since the sensors in microelectromechanical systems can have a small size, low weight, low power consumption, low cost, potential for use in special places, potential for integration on a chip, and increased efficiency and high reliability, the measurement systems developed based on this technology have garnered more attention in recent years. However, the measurements by these sensors are affected by errors. Hence, to improve the precision and accuracy of the measurements, these sensors (accelerometers and magnetometers) require calibration to provide an accurate estimate of the positions based on the output components of the sensors in directional drilling and carefully control the well drilling path towards an undetermined target. This paper concentrates on the calibration of the accelerometer sensors of microelectromechanical systems.

According to the scientific articles published in recent years, numerous studies have been carried out to calibrate the accelerometer and magnetometer sensors of microelectromechanical systems. Unfortunately, very few of these studies have addressed the sensors used in measurement while-drilling systems as stated in the following. For instance, in (Yang et al., 2013), a strong inclinometer is developed using three microelectromechanical systems single axis accelerometers and three barometer sensors, which are calibrated by formulating a sensitive tri-axis sensor linear model and determining azimuth and inclination. Two different optimum solutions, namely the internal reflective newton and sequential quadratic programming (SQP) methods were proposed to reduce the tilt and azimuth system errors and improve the accuracy of the proposed model. In (Qian et al., 2011), a design is proposed to sense tilt using a physical model composed of three microelectromechanical systems accelerometers. Besides, three numerical methods are developed for sensing the inclination. Afterward, a bias model is introduced to reduce the error resulting from the nonlinear relationship between the gravitational acceleration and inclination via a nonlinear relation. All of these three numerical models are consolidated into a linear model whose parameters can adequately be estimated using the least squares error (LSE) method. It has also been proven that this design can sense tilt with minor error. Moreover, a new solution for calibration of tri-axis microaccelerometers is proposed in (Frosio et al., 2009) based on the fact that under static conditions the absolute value of the accelerometer output vector has to comply with the gravitational acceleration. This solution is equipment-independent, and this model determines the calibration of the bias and scale factor for each axis as well as the mutual axes. The model parameters are calculated through Newton's method in nonlinear optimization, revealing that the sensor output calculated via this type of calibration is more accurate than the calibration by the manufacturer and other conventional calibration methods. Article (Ang, Khosla and Riviere, 2007) presents a nonlinear regression model for the capacitive accelerometers of microelectromechanical systems that can be used to sense tilt and meet low-acceleration motion tracking purposes. The model proposed for the accelerometer deterministic errors (the bias, scale factor, and misalignment errors) is used to calibrate the accelerometer. The proposed model lowered the sensor errors to the residual random noise level. The authors of (Yanshun, Shuwei and Jiancheng, 2012) proposed a measurement while-drilling (MWD) tool based on the inertial measurement unit. Their tool consisted of a fiber optic gyroscope (FOG) and an accelerometer and improved the accuracy and precision of inertial sensors with the aim of improving the precision of measurement of the state angles (azimuth and pitch). In (Aydemir and Saranlı, 2012), the researchers reviewed the deterministic errors and sources of random noise for the inertial sensors of the microelectromechanical systems and proposed a calibration procedure for the inertial measurement (composed of an accelerometer and a magnetometer) of the inertial navigation system to develop models suiting these errors. In (Wei et al., 2013), a mathematical model is analyzed based on the properties of the inertial sensors errors in microelectromechanical systems and the validity of the six-state method is confirmed using a tri-axis magnetometer and accelerometer. The errors of installation, bias, and scale factor in the inertial sensors of microelectromechanical systems are also fixed. The authors of (Bonnet et al., 2009) present a calibration framework to increase sensor accuracy and determine the precise state in the inertial navigation systems. The sensor calibration framework is formulated in a goodness of fit problem and different calibration parameters such as sensitivity, offset, and the misalignment angle are determined. In (Camps, Harasse and Monin, 2009), it is stated that the accelerometer and magnetometer sensor parameters have to be estimated precisely to prevent drift. Hence, calibration is an important step in the correction of the use of these sensors and calculation of the expected measurements in terms of the inertial measurement unit.

This paper presents the experimental and theoretical steps of a numerical calibration method for the calculation of the gain, bias, and nonorthogonality of the magnetometer and accelerometer sensors. In (Die, Chunnian and Hong, 2011), it is stated that the single axis microelectromechanical accelerometers provide higher transparency than tri-axis accelerometers, and it is tried to propose a more accurate method of calibrating these accelerometers. The results of both calibrations carried out by the conventional six-parameter method and the new 12-parameetr method are also compared using the tri-axis desk as the reference. The results of the comparison suggest that the accuracy of the measurement resulting from the 12-parameter calibration method is higher than the accuracy of the 6-parameter calibration methods. In (Marinov and Petrov, 2014), a low-cost solution for the static calibration of an MEMS accelerometer is proposed to estimate the accelerometer mathematical model consisting of the scale factor and bias errors. This method does not require additional equipment items and entails simple calculations. However, the MWD system has been studied in very few of the mentioned articles. In some of these articles, angle calibration is carried out instead of acceleration calibration to obtain the sensor parameters.so, there is a need for the angle data. In some other articles, Newton's method in optimization is utilized. This method revolves around differentiation and is time-consuming. It is also highly sensitive to the initial values and requires the adjustment of the damping ratio. In other studies, the sensitivity and misalignment matrix is considered to be symmetrical. These studies use a simple 9-parameter model instead of a 12-parameter model. Therefore, their models are not adequately precise, or a combination of the least squares error method and the six-state method is used, which still lacks the required precision and accuracy.

Measurement While Drilling Tools

The measurement while-drilling (MWD) tools transmit real-time logging information to the surface, enabling the driller to practice real-time control over the directional drilling path of the well. In the measurement while-drilling tools, the battery module or generator supplies power to the system. The directional module (including the tri-axis accelerometers and tri-axis magnetometers and the microprocessors that process the information) obtains the positional information including the inclination, the azimuth or direction, and the tool face angle of the well. The accelerometers function to measure the gravitational acceleration and determine the deviation from the vertical path of the well while the magnetometers serve to measure the earth's magnetic field to find the direction. A combination of the magnetometers and accelerometers is also used to measure azimuth. The gamma radiation sensor module identifies the rock materials and different strata and the pulser is in charge of transmitting the modulated information to the surface. Figure (1) depicts a directional module.

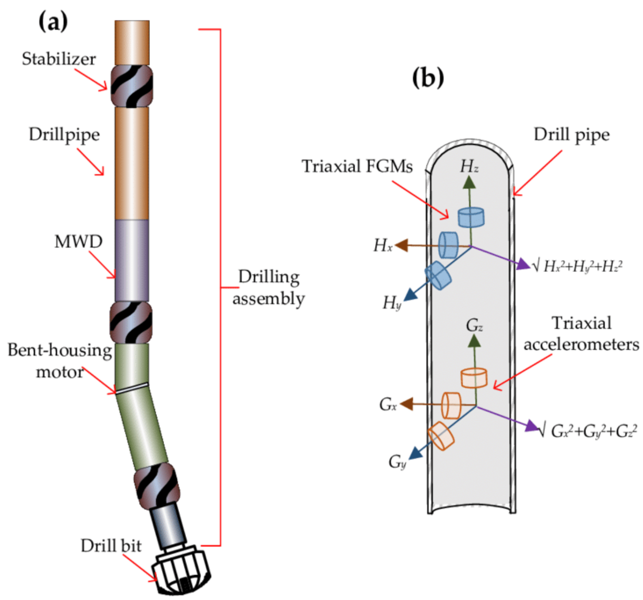

Figure 1: The directional module including the accelerometer and magnetometer in the measurement while-drilling tool of a drilling string

Inclination: It is the angle between a vertical line and the drilling path (the logging tool) of the well at any point. This angle varies between 0 and 180 degrees. The maximum allowable error for this angle is 0.25 degrees, which is calculated via the following relation based on the acceleration components. Moreover, this angle, which is denoted by ρ , is known as inclination, deviation, tilt, and pitch.

$$I = arc\,tan\frac{\sqrt{g_x^2 + g_y^2}}{g_z} \tag{1}$$

Azimuth: It is the angle between the magnetic north reference (or the actual north reference) and the projection of the well drilling path (the logging tool) on the horizontal plane in the clockwise direction. This angle varies between 0 and 360 degrees. The maximum allowable error for this angle is 2-3 degrees, which is calculated via the following relation using the acceleration and magnetic components. This angle, which is denoted by φ , is known as azimuth, rotation, and roll.

$$A = arc\,tan\frac{(m_y g_x - m_x g_y)g}{m_z(g_x^2 + g_y^2) - g_z(m_x g_x + m_y g_y)} \tag{2}$$

Tool face angle: It is the rotation angle of the drilling bit. This angle varies between 0 and 360 degrees.

Microelectromechanical Acceleration Sensor Errors

- **Deterministic Errors (Systematic, Fixed, or Inherent) of the Acceleration Sensor**

Given the micro dimensions of these sensors, it is difficult to manufacture sensors with identical characteristics as the slightest impurity can considerably change the results. The deterministic or systematic errors are caused by the manufacture technology and the defects in the manufacturing process. These errors are identified through survey and compensation and are ruled out from the data. The dominant systematic errors are as follows:

- **Bias (Offset)**: It is the average observed output in the course of a given period when no input is applied to the accelerometer. The bias of an ideal sensor is zero.
- **Scale factor (sensitivity):** It is the ratio of the variations of the sensor output signal to the variations of the measured input. An ideal sensor has a scale factor of one.
- **Misalignment or nonorthogonality error**: The nonorthogonality error is the error caused by the incorrect installation of the sensors at the time of manufacture. Moreover, there might be the misalignment error in the sensitive axes of the tri-axis accelerometers and the orthogonal axes of the object body.
- **Dependence of bias and scale factor on temperature**: this error reflects the variations of bias and the scale factor with sensor temperature. These variations, which are compensated, are estimated by calculating the dependence of the sensor parameters on temperature under operating conditions at different temperatures. The effect of temperature on bias is also larger than the effect of scale.

- **Random (Stochastic) Errors of the Acceleration Sensor**

The accelerometer random error includes the noises especially white noises, which disrupt the performance of the sensor. This noise is about several thousandths of the gravitational acceleration. White Gaussian noise has the normal probability distribution with a zero mean ($\mu = 0$) and unit variance ($\sigma = 1$). Therefore, it is omitted by averaging the data.

The Mathematical Model of the Microelectromechanical Accelerometer and Its Parameters

A suitable sensor has high sensitivity, low offset, no misalignment, no dependence on temperature, and little noise. As seen in the block diagram depicted in Figure (2), the acceleration model is extracted as follows.

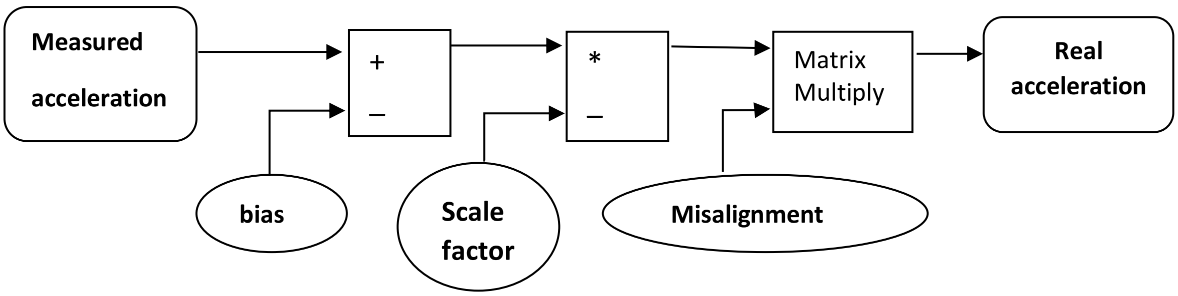

Figure 2: The accelerometer model block diagram

Consider $O = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix}$ be the bias (offset) vector, $S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$ be the scale matrix (sensitivity), and $c_b^m = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} = \begin{bmatrix} \cos\varphi_{x'x} & \cos\varphi_{x'y} & \cos\varphi_{x'z} \\ \cos\varphi_{y'x} & \cos\varphi_{y'y} & \cos\varphi_{y'z} \\ \cos\varphi_{z'x} & \cos\varphi_{z'y} & \cos\varphi_{z'z} \end{bmatrix}$ be the matrix for the conversion of the coordinates between the three sensitive accelerometer sensor axes and the orthogonal axes of the body of the object (with φ denoting the angle between three axes of accelerometer sensor and the orthogonal axes of the object body). Besides, consider $n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ be the noise vector, $\tilde{A}^T = [\tilde{a}_x \quad \tilde{a}_y \quad \tilde{a}_z]$ be the normalized measured acceleration output vector, and $A^T = [a_x \quad a_y \quad a_z]$ be the normalized ideal acceleration output vector, where $\tilde{a}_x$, $\tilde{a}_y$, and $\tilde{a}_z$ represent the measured values of the acceleration components, a_x, a_y, and a_z represent the ideal values of the acceleration components, o_x, o_y, and o_z denote the elements of the bias matrix O, and s_x, s_y, and s_z are the elements of the scale factor matrix, S. Therefore, the MEMS accelerometer system error model is written as follows:

$$\tilde{A} = S \times C_b^m \times A + O + N$$
$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad (3)$$

By overlooking the noise vector and combining the sensitivity and nonorthogonality matrices we have:

$$\tilde{A} = S_t \times A + O$$
$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \begin{bmatrix} s_x M_{xx} & s_x M_{xy} & s_x M_{xz} \\ s_y M_{yx} & s_y M_{yy} & s_y M_{yz} \\ s_z M_{zx} & s_z M_{zy} & s_z M_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \quad (4)$$

The following relation is obtained by inversing the relation above.

$$A = S_t^{-1} \times (\tilde{A} - O)$$
$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} s_x M_{xx} & s_x M_{xy} & s_x M_{xz} \\ s_y M_{yx} & s_y M_{yy} & s_y M_{yz} \\ s_z M_{zx} & s_z M_{zy} & s_z M_{zz} \end{bmatrix}^{-1} \left(\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} - \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \right) \quad (5)$$

Finally, the accelerometer model is written as follows after simplification.

$$A = S_{tnew} \times (\tilde{A} - O)$$
$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} s_{xx} & s_{xy} & s_{xz} \\ s_{yx} & s_{yy} & s_{yz} \\ s_{zx} & s_{zy} & s_{zz} \end{bmatrix} \left(\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} - \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \right) \quad (6)$$

Where $S_{tnew} = \begin{bmatrix} s_{xx} & s_{xy} & s_{xz} \\ s_{yx} & s_{yy} & s_{yz} \\ s_{zx} & s_{zy} & s_{zz} \end{bmatrix}$ is a matrix consisting of the scale factor errors along the three axes (diagonal entries) as well as their misalignment (the non-diagonal entries).

In an ideal accelerometer, the factors of the facing axes are zero, but they can be as large as 2% of the sensor sensitivity in real accelerometers. By overlooking the error of misalignment during installation, the sensitivity matrix transforms into a diagonal matrix. According to this model, the actual acceleration values are obtained if the bias vector values are subtracted from the measured values and the product is multiplied by the scale matrix.

Six-State Static Test and MEMS Accelerometer Calibration

In experimental calibration, a six-state static test (the top and bottom states for the three axes) is carried out to collect the accelerometer outputs. Only the bias error values and the bias factors are obtained from this test. If the sensor is a tri-axis sensor, the nonorthogonality error can be overlooked with high reliability among the other errors. To this end, first each accelerometer axis is placed exactly along the gravitational acceleration at constant temperature and the output is recorded. Afterward, it is rotated 180 degrees (in the direction opposite to the gravity direction) and the output is recorded again. To eliminate the noise effect, measurements are repeated and the average value is calculated. Table (1) presents the method of this experiment.

Table 1: The six-state test

ID	Stationary position	a_x	a_y	a_z
1	Z down	0	0	**+1g**
2	Z up	0	0	**-1g**
3	Y down	0	+1g	**0**
4	Y up	0	-1g	**0**
5	X down	+1g	0	**0**
6	X up	-1g	0	**0**

Since the output of the accelerometer sensor (the local acceleration components) is affected by temperature (thermal noise) and the environmental conditions in general (The temperature grows one degree for every 100-meter increase in depth.), the calibration carried out by the manufacturers does not meet the engineering requirements. Hence, recalibration of the accelerometer is necessary to reduce the deterministic error (the scale factor/bias/misalignment error). In this article, two calibration methods are proposed to calibrate accelerometers and estimate the parameters of the scale factor matrix (sensitivity) for each axis and the symmetrical axes (misalignment) (s_x s_y s_z s_{xy} s_{xz} s_{yx} s_{yz} s_{zx} s_{zy}) and The bias (offset) of the three axes (o_x o_y o_z) based on the outputs of the microelectromechanical systems accelerometer, and the advantages and disadvantages of each of them are explained subsequently. It is possible to have more realistic estimates using the sensor model and its scale factor and bias values as well as measured acceleration values. In this case, the inclination and azimuth values calculated based on the acceleration components are more realistic and the errors are reduced. It is also worth noting that in this paper, the normalized acceleration values resulting from the roll test carried out by Sealand Company are used. The MWD tool is PDT. The gravitational acceleration naturally varies by latitude and elevation (from 9.78 $\frac{m}{s^2}$ on the equator to 9.83 $\frac{m}{s^2}$ in the poles). The average value also almost equals g=9.81 $\frac{m}{s^2}$. The g value is calculated as follows using the international gravity.

$$g = 9.7803327[1 + 5.3024 \times 10^{-3}\sin^2\theta - 5.8 \times 10^{-6}\sin^2\theta] \pm 3.086 \times 10^{-6}h \quad (7)$$

A reliable g value expressed in terms of $\frac{m}{s^2}$ is a function of latitude θ and elevation h. However, since with a 10000ft (3048m) increase in depth, the gravitational acceleration decreases by 0.0005g, which is insignificant, the 9.81 $\frac{m}{s^2}$ value is used as the normalized gravitational acceleration of 1 similar to the majority of the articles. Hence, the input data, which are the acceleration components (g_z, g_y, and g_x), have normalized values varying between 0 and 1.

Accelerometer Calibration Using the Least Squares Error Method and the Particle Swarm Optimization Algorithm

If $A = [a_x \quad a_y \quad a_z]^T$ is the actual acceleration vector, $\tilde{A} = [\tilde{a}_x \quad \tilde{a}_y \quad \tilde{a}_z]^T$ is the measured acceleration vector, $O = [o_x \quad o_y \quad o_z]^T$ is the accelerometer bias vector, and $C_{ms} = \begin{bmatrix} S_x M_{xx} & S_x M_{xy} & S_x M_{xz} \\ S_y M_{yx} & S_y M_{yy} & S_{yz} M_{yz} \\ S_z M_{zx} & S_z M_{zy} & S_z M_{zz} \end{bmatrix}$ is the matrix of the scale factor and misalignment errors, then the following relation is obtained based on the accelerometer model and relation $\tilde{A} = C_{ms} A + B$.

$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \begin{bmatrix} s_x M_{xx} & s_x M_{xy} & s_x M_{xz} \\ s_y M_{yx} & s_y M_{yy} & s_y M_{yz} \\ s_z M_{zx} & s_z M_{zy} & s_z M_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \quad (8)$$

The accelerometer model is rewritten as follows:

$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} = \begin{bmatrix} s_x M_{xx} & s_x M_{xy} & s_x M_{xz} & o_x \\ s_y M_{yx} & s_y M_{yy} & s_y M_{yz} & o_y \\ s_z M_{zx} & s_z M_{zy} & s_z M_{zz} & o_z \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \\ 1 \end{bmatrix} \quad (9)$$

Therefore,

$$x = \begin{bmatrix} s_x M_{xx} & s_x M_{xy} & s_x M_{xz} & o_x \\ s_y M_{yx} & s_y M_{yy} & s_y M_{yz} & o_y \\ s_z M_{zx} & s_z M_{zy} & s_z M_{zz} & o_z \end{bmatrix} \tag{10}$$

$$\tilde{a} = \begin{bmatrix} \tilde{a}_{x1} & \tilde{a}_{x2} & \dots & \tilde{a}_{xn} \\ \tilde{a}_{y1} & \tilde{a}_{y2} & \dots & \tilde{a}_{yn} \\ \tilde{a}_{z1} & \tilde{a}_{z2} & \dots & \tilde{a}_{zn} \end{bmatrix} \tag{11}$$

$$a = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6]$$

$$a_1 = \begin{bmatrix} g \\ 0 \\ 0 \\ 1 \end{bmatrix} a_2 = \begin{bmatrix} -g \\ 0 \\ 0 \\ 1 \end{bmatrix} a_3 = \begin{bmatrix} 0 \\ g \\ 0 \\ 1 \end{bmatrix} a_4 = \begin{bmatrix} 0 \\ -g \\ 0 \\ 1 \end{bmatrix} a_5 = \begin{bmatrix} 0 \\ 0 \\ g \\ 1 \end{bmatrix} a_6 = \begin{bmatrix} 0 \\ 0 \\ -g \\ 1 \end{bmatrix}$$

The following relation is obtained through the six-state test and the least squares method.

$$\rightarrow x = \tilde{a}.a^T.(aa^T)^{-1}\tilde{a} = x * a \tag{12}$$

The resulting matrix, x, is a three-row and four-column matrix. The first three columns of this matrix represent the elements of the scale factor matrix, and the fourth column represents the elements of the bias vector. This technique estimates the bias, scale factor, and misalignment errors for the x, y, and z axes using the measurements resulting from six states.

The bias and scale factor matrix resulting from this method is presented in Table (2) for the accelerometer model, which also estimates the actual acceleration values.

Table 2: The bias and scale matrix resulting from the least squares method

Sensivity or scale matrix	Offset or bias matrix
$S = \begin{bmatrix} 1.0011 & 0.0421 & 0.0109 \\ -0.0378 & 1.0060 & 0.0203 \\ 0.0034 & 0.0008 & 0.9980 \end{bmatrix}$	$\mathbf{B} = \begin{bmatrix} \mathbf{-0.0041} \\ \mathbf{0.0071} \\ \mathbf{-0.0027} \end{bmatrix}$

It is possible to calibrate the acceleration sensor and obtain the model parameters (the scale factor and bias vector matrix) (position) by conducting an optimization algorithm and defining the objective or cost function as the difference between the estimated acceleration value and the ideal acceleration value by minimizing the cumulative error, E, in relation to the parameters.

$$E = \frac{1}{N}\sum_{i=1}^{N} e_i^2$$
$$e = \sqrt{(\bar{a}_x - \tilde{a}_x)^2 + (\bar{a}_y - \tilde{a}_y)^2 + (\bar{a}_z - \tilde{a}_z)^2} \tag{13}$$

In these relations, $\bar{a}$ is the ideal acceleration and $\tilde{a}$ is the estimated acceleration. Moreover, i denotes the experiment number. The stop condition can be the achievement of an acceptable solution or the acceptable number of iterations of the objective function. Here, the number of iterations of the algorithm for obtaining the desirable solution (MaxIt) equals 600 and the number of particles randomly distributed in the problem search space or the size of the initial particle population (nPop) is usually at most 10 times the number of variables, which equals 60. This method is implemented within less than 10 seconds.

By meeting the objective function by minimizing the difference between the measured and the ideal acceleration components in six states, the bias and scale matrix for the accelerometer model, which estimates the actual acceleration values, is written in Table (3).

Table 3: The bias and scale matrix resulting from the optimization algorithm

Sensivity or scale matrix	Offset or bias matrix
$S=\begin{bmatrix} 1.0011 & 0.0421 & 0.0108 \\ -0.0378 & 1.0060 & 0.0202 \\ 0.0034 & 0.0008 & 0.9979 \end{bmatrix}$	$B=\begin{bmatrix} \mathbf{0.0041} \\ \mathbf{-0.0071} \\ \mathbf{0.0027} \end{bmatrix}$

The errors resulting from several experiments in different methods are depicted in Figure (3). This figure shows the contribution of each calibration method to decrease the error between the estimated acceleration values and the ideal acceleration values. As seen, both methods reduced the errors. However, the least squares error method resulted in errors in one experiment, whereas the optimization algorithm reduced the errors satisfactorily.

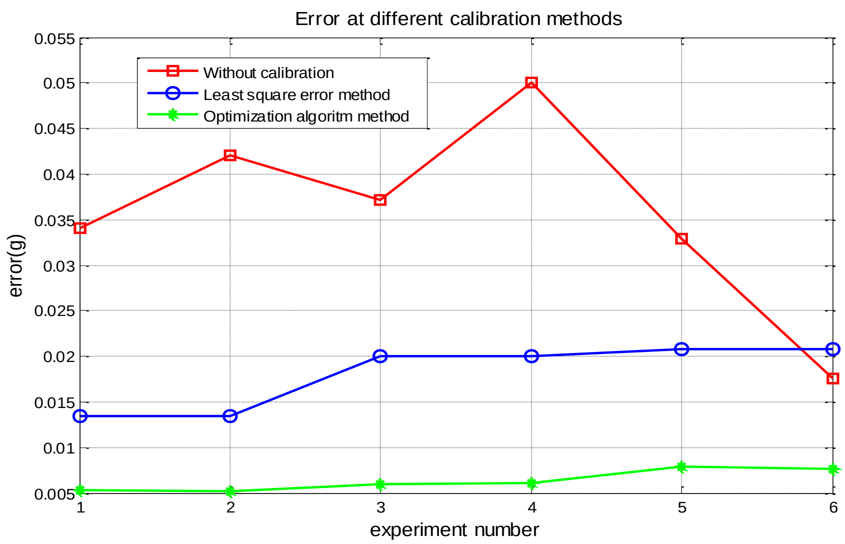

Figure 3: The comparison between the errors in different methods in several tests

based on the acceleration error, which is defined as $e=\sqrt{(\bar{a}_x-a_x)^2+(\bar{a}_y-a_y)^2+(\bar{a}_z-a_z)^2}$ after calibration and $e=\sqrt{(\bar{a}_x-\tilde{a}_x)^2+(\bar{a}_y-\tilde{a}_y)^2+(\bar{a}_z-\tilde{a}_z)^2}$ before calibration (where $\bar{a}$ is the ideal acceleration, $\tilde{a}$ is the measured acceleration, and a is the estimated acceleration).

Based on these values, the mean $(\bar{e}=\frac{1}{N}\sum_{i=1}^{N}e_i)$, standard deviation $(\sigma=\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(e_i-\bar{e})^2})$, mean squared error $(MSE=\frac{1}{N}\sum_{i=1}^{N}e_i^2)$, and the root mean squared error $(RMSE=\sqrt{\frac{1}{N}\sum_{i=1}^{N}e_i^2})$ are calculated and the model accuracy is assessed. The assessment of different methods against the mean, standard deviation, mean squared error, and root mean squared error criteria (effective error) is presented in Table (4). As seen, there was a post-calibration improvement and the calibration performance was improved using the optimization algorithm as compared to the conventional least squares method.

Table 4: The assessment of the different calibration methods

	MSE	RMSE	Mean	STD
without calibration	0.0014	0.0370	0.0356	**0.0108**
Calibration with least square error method	$3.3823 * 10^{-4}$	0.0184	0.0181	**0.0037**
Calibration with optimization algorithm method	$4.1819 * 10^{-5}$	0.0065	0.0064	**0.0012**

Conclusion

In this paper, the errors in an accelerometer were studied and modeled. The model was proposed for the systematic accelerometer errors including the bias, scale factor, and misalignment errors. Measurements were carried out several times and the average value was calculated to eliminate the effect of noise. Moreover, the effect of temperature on the aforesaid parameters was studied. Suitable methods for the calibration of these sensors and attainment of the model parameters using the sample data extracted from the MWD (measurement while-drilling) tools were proposed and the system output acceleration was calculated using the proposed model. Following the calibration process, the estimated acceleration values were similar the ideal values more than the measured values and the error decreased. It was also found out that the accelerometer calibration through the optimization algorithm and minimization of the squared error of the sensor output and the ideal values showed a higher level of accuracy than the conventional least squares method.

The aforementioned procedure can be utilized to analyze the magnetometer error with slight changes in the process of modeling the accelerometer error and calibration. Besides, by defining a magnetometer model along with an accelerometer model, it is possible to rewrite the error as the tilt and azimuth error and calculate the model parameters by minimizing the tilt and azimuth error to carry out another form of calibration, which is angle calibration. Due to the thermal variations in directional drilling and the dependence of the proper performance of the sensors on the environmental conditions, especially temperature (the offset/bias and sensitivity/scale factor are temperature-dependent), the thermal compensation and elimination of the error of the bias and scale factor variations resulting from the temperature variations through polynomial fitting (interpolation of the relationship between temperature and the bias and scale factor values) for the three accelerometer axes are necessary. In general, numerous random noise and error components exist in the data depending on the tool and the environment producing the data. Using the Allan variance (AVAR) or other techniques, it is possible to model and estimate the sensor output stochastic behavior and obtain a more accurate model. Here, the experiments were repeated several times under identical conditions to remove the noise from the measurements. Finally, since the 12-state method offers more states than the 6-state method, it definitely improves the accuracy of the results if the tests are feasible.

References

1. Ang, W. T., Khosla, P. K., and Riviere, C. N.2007.Nonlinear Regression Model of a Low-g MEMS Accelerometer.IEEE Sensors Journal., VOL. 7 : 81-88.
2. Aydemir, G. A., and Saranlı, A. 2012. Characterization and calibration of MEMS inertial sensors for state and parameter estimation applications. Measurement., Vol.45 :1210-1225.
3. Bonnet, S., Bassompierre, C., Godin, C., Lesecq, S., and Barraud, A. 2009. Calibration methods for inertial and magnetic sensors. Sensors and Actuators., Vol.156: 302-311.
4. Camps, F., Harasse, S., and Monin, A. 2009. Numerical calibration for 3-axis accelerometers and magnetometers. Electro/Information Technology. IEEE International Conference, 217-221.
5. Die, H., Chunnian, Z., and Hong, L. 2011. Autocalibration method of MEMS accelerometer. Mechatronic Science, Electric Engineering and Computer (MEC). IEEE International Conference ., 1348-1351
6. Frosio, I., Pedersini, F., and Alberto Borghese, N. 2009. Autocalibration of MEMS accelerometers.Instrumentation and Measurement. IEEE Transactions., Vol. 58 : 2034-2041.

7. Marinov,M., and Petrov, Z.2014. An approach for static calibration of accelerometer MMA8451Q.Scentific research and education in the air force-AFASES.
8. Qian, J., Fang, B., Yang, W., Luan, X., and Nan, H. 2011. Accurate tilt sensing with linear model. Sensors Journal. IEEE.,Vol. 11: 2301-2309.
9. Wei, R., Tao, Z., Hai-yun, Z., Lei-gang, W., Yong-jie, Z., Meng-kai, L., and Jing-wei, S. 2013. A research on calibration of low-precision MEMS inertial sensors. Control and Decision. IEEE Conference ., 3243-3247.
10. Yang, W., Fang, B., Tang, Y. Y., Qian, J., Qin, X., and Yao, W. 2013. A robust inclinometer system with accurate calibration of tilt and azimuth angles.Sensors Journal.IEEE., Vol. 13: 2313-2321.
11. Yanshun, Z., Shuwei, W., and Jiancheng, F. 2012. Measurement-while-drilling instrument based on predigested inertial measurement unit. Instrumentation and Measurement. IEEE Transactions on.,Vol. 61: 3295-3302.