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A Fuzzy Goal Programming Approach to Multi-Objective Optimization for Interdependent Information System Project Selection

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Abstract: *Project selection problems of Information System (IS) are multi-criteria decision-making (MCDM) problems that usually there are interdependencies among their criteria and candidate projects and to know these interdependency is very important in decision making for decision maker. Among the existing methodology of multi-criteria decision making (MCDM), Goal Programming (GP) is widely used for IS project selection. GP instead of the direct evaluation of criteria outcomes models explicitly the desired target value for each criterion and optimize the deviations of criteria outcomes from these goals. The solution depends on the metrics for the deviations and as well as the waiting method of the different goals. In order to provide a systematic approach to set priority among multi-criteria and tradeoff among objectives, Analytic Network Process (ANP) is suggested to be applied prior to GP formulation. With use of ANP we get to the priority among objectives then this method can generate the results that consistent with the decision maker expectation that the goal with higher priority may have higher level of satisfaction. Since a decision is usually vague, it may be based on fuzzy number.*

Keywords: *Information System (IS), Project Selection, Fuzzy Goal Programming (FGP) Approach, Multi Objective Decision Making (MODM); Analytic Network Process (ANP)*

INTRODUCTION

Decision making processes in real-world especially in business has become one of the most important fields. Decision processes with multiple criteria deal with human judgment and is not easy to model.
Evaluation and selection of information system (IS) project is concerned with allocation of scarce organization resources. Various method have been proposed to help organizations make good IS project selection decisions. At Information system (IS), projects are evaluated according to different criteria and with considering interdependency among criteria and candidate projects saves valuable cost in organizations and the problem was made closer to real world problems. Also, in reality it will be more appropriate to consider multiple criteria than to consider only one or two criteria in IS project selection problems with interdependence property.
In order to solve optimization problems, many researchers use a mathematical model, such as goal programming, dynamic programming, linear 0-1 programming and other suitable methods.

Moreover, social responsibility is one of the critical goals that organizations should be consider ed. Pressures from the NGOs, social communities and media to respect social issues caused a lot of damages to some of well -known corporations. orporate Social Responsibility (CSR) deals with the effect of corporate activities on different social entities such as job opportunity, human rights, labor safety, etc. (Li X, et al., 2016).

The GP approach of multi criteria problems has received increasing interest due to its modeling flexibility and conceptual simplicity. GP instead of the direct evaluation of criteria outcomes models explicitly the desired target value for each criterion and optimize the deviations of criteria outcomes from these goals.

Jin and Soung (Jin Woo Lee, et al., 1998) suggested an IS project selection which reflected interdependencies among evaluation criteria and candidate project using ANP but they didn't have any attention to the uncertainty of numbers. Since a decision is usually vague, it maybe based on fuzzy numbers. In GP the preferences required from the decision maker are presented with weights, targets, tradeoffs and goal levels to formulate the problem.

According to the fuzzy theory, the inaccurate objectives and constraints are represented by certain kind of membership function, for instance, the triangle-like or trapezoid-like membership function, we call the inaccurate objectives and constraints as fuzzy objectives and constraints.

After presentation of framework of fuzzy decision by Bellman and Zadeh (Bellman R.E, et al., 1970) Zimmerman (1978) utilized Fuzzy Programming (FP) approach to linear programming with several objectives (Zimmerman H.J, 1978). In general, FP is a GP with some weights assigned to the deviational variables in the objective function, where the FP has fuzziness in the aspiration level, i.e. to get a solution that makes the objectives as close as possible to a specific goal within a certain limit.

In GP problems, it is important and practical to consider different priorities for goals. for make these priority we can use from weighting method for example AHP or ANP.

The basic assumptions of AHP are that it can be used in functional interdependence of an upper part or cluster of the hierarchy from all its lower parts and the criteria or items in each level. Then much decision making cannot be structured hierarchically because they involve the interaction and dependence of higher level element on a lower level element. in the result of this limitation Saaty (Saaty TL, 1980) had comprehensive study and he suggested the use of AHP to solve the problem of independence of on alternatives or criteria and use of Analytical Network Process (ANP) for a network system that functional dependence allows feedback among clusters. The most important function of ANP is to determine the relationship of a network structure or degree of interdependence.

The ANP addresses how to determine the relative importance of a set of activities in a multi criteria decision problem. In order to solving of interdependence IS project selection, first we should identify the multiple criteria that merit consideration and then draw a relationship between criteria that shows the degree of interdependence among the criteria.

After providing the actual weights by use of ANP, goal priority for ever object is identified.

In this paper, we present by using analytic network process we provide weight of projects and then priority of objectives that they use in methodology to solve of the fuzzy goal programming approach to multi objective optimization for IS project selection problems that have multiple criteria and interdependence property among criteria and alternatives.

Literature Review

The existing methodologies for IS project selection range from single _criteria cost / benefit analyst to multiple criteria scoring model and ranking methods, or subjective committee evaluation methods. Buss presented an alternative approach to project selection with ranking technique. Lootsma et al. and Lucas and Moore suggested a multiple-criterion scoring method for IS project selection (Jin Woo Lee, et al., 1998). Muralidhar and Wilson proposed a methodology for IS project selection using AHP (Jin Woo Lee, et al., 1998). Ranking, scoring and AHP methods don't apply to problems having resource feasibility, optimization requirement or project interdependence property constraint. In spite of these limitations, these methods have been much used with real problems because they are very simple and easy to understand, so decision maker feel comfortable with them. Santhanam and Kyparisis proposed a mathematical methodology using nonlinear 0-1 programming for interdependent information system selection but they considered project selection problems that have only one criterion not multiple criteria (Jin Woo Lee, et al., 1998).

The initial study identified the multi decision technique known as the Analytic Hierarchy Process (AHP) to be the most appropriate for solving complicated problems. This method was proposed by Saaty in 1980 for solving socio – economic decision making problems and solved a wide range of problems (Saaty TL., 1980)

Goal programming (GP) that firstly introduced by Charnes and Cooper in the early 1960's, is a useful method for decision maker to consider simultaneously many goals for satisfactory solution. First attempts to model decision processes with multiple criteria in business lead to concepts of goal programming (Ignizio, 1976).

This method is a robust tool for multi objective decision making (MODM) problems and has been studied for upper than 35 years.

Jin and Soung (Jin Woo Lee, et al., 1998) presented a methodology using analytic network process and zero-one goal programming for IS project selection but their research limited to script modeling. While determine of goal value of each objective is difficult for DM to incorporate uncertainty and imprecision into the formulation, the fuzzy set theory, initially proposed by Zadeh in 1965 is introduced in the field of conventional MODM problems where aspiration level of objectives are assigned in an imprecise manner. Lai and Hwang see the application of over simplified (crisp) models such as goal programming. Fuzzy multi criteria models are robust and flexible. Decision maker consider the existing alternatives under given constraints but they also develop new alternatives by considering all possible situations (Lai & Hwang, 1995).

Zadeh (1965) initiated the fuzzy set theory. and Zadeh (1970) presented some applications of fuzzy theories to the various decision-making processes in a fuzzy environment. Zimmermann (1976, 1978) presented a fuzzy optimization technique to linear programming (LP) problem with single and multiple objectives. Narsimhan (1980) proposed a fuzzy goal programming (FGP) technique to specify imprecise aspiration levels of the fuzzy goals. Tiwari, Dharmar, and Rao (1986) presented a simple additive model to formulate an FGP problem. Yang, Ignizio, and Kim (1991) later formulated the FGP with nonlinear membership functions by using the min operator for aggregating goals to maximize the decision set. Hannan considered the same FGP problem as Narasimhan (Lachhwani; 2008) and he developed a model to solve it (Chen, H.K., 1994) Both Narasimhan and Hannan consider fuzzy goals with isosceles triangular linear membership functions (TLMFs). Kim and Whang have attempted to improve the Hannan and Narasimhan models. They propose a model for solving an FGP problem that can handle fuzzy goals with unbalanced (non-isosceles, asymmetric) TLMFs. Unbalanced TLMFs often appear in quality, inventory, personnel management and economics. However, in a recent note , it is shown that more constraints should be added to Kim and Whang model, otherwise it might yield undesirable results in comparison with the Hannan and Narasimhan models. Another model which could deal with unbalanced TLMFs was introduced by Yang et al.

Sonja and Ajith (Sonja Petrovic, et al.) worked on fuzzy MCDM and compared it with non fuzzy MCDM. Fuzzy MCDM is a good approach for solving model but applying goal for each of object help receive to optimized solution easier and faster.

In this paper, we make feasible space for solving the fuzzy goal programming with priority by use of fuzzy MCDM and for decrease of decision maker's responsibility, we made quantity priorities for objects then it's enough that decision maker appoints value of any goal.

No prior study reported in the literature has ever demonstrated the solving methodology of an IS project selection that have both multiple criteria and interdependence property with fuzzy data.

Proposed Fuzzy Goal Programming Framework to Multiple Objective IS Project Selection with Multiple Priority Using Analytic Network Process (ANP) To Achieve Linear Weighs Associated to Each Project

As described in later sections, when we deal with interdependent project selection problems, we have to apply ANP to find linear weights. Calculated weights in this step (wj) will be used to formulate mathematical model in next steps.

Developed FANP is illustrated by a numerical example given in section4 of this paper.

The Fuzzy Goal Programming Model

Fuzzy set theory uses linguistic variables rather than quantitative variables to represent imprecise concepts. Linguistic variables exhibit the vagueness of human language.

Fuzzy set: Let X be a universe of discourse, A is a fuzzy subset of X if for all xεX, there is a number $\mu_A(x)$ ε [0,1] assigned to represent the membership of x to A, and $\mu_A(x)$ is called the membership function of A.

Fuzzy number: Among the various types of fuzzy sets, of special significance are fuzzy sets that are defined on the set R of real numbers. Membership functions of these sets, which have the form $A: R \rightarrow [0,1]$ clearly have the quantitative meaning and may, under certain conditions, be viewed as a fuzzy numbers or fuzzy intervals. To qualify as a fuzzy number, a fuzzy set A on R must possess at least the following three properties:

1) A must be a normal fuzzy set;

2) A_α must be a closed interval for every $\propto \in (0,1]$;

2) A_α must be a closed interval for every $\propto \in (0,1]$;

3) The support of A must be bounded.

Triangular fuzzy number: In a universe of discourse X, a fuzzy subset A of X is defined by a membership function $f_A(x)$, which maps each element x in X to a real number in the interval [0, 1]. The function value $f_A(x)$ represents the grade of membership of x in A.

A fuzzy number A (Hannan, E.L., 1981) in real line R is a triangular fuzzy number if its membership function $f_A: \Re \rightarrow [0,1]$ is

$$f_A(x) = \begin{cases} (x-a)/(b-a) & a \leq x \leq b \\ (x-c)/(b-c) & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

With $-\infty < a \leq b \leq c < \infty$. The triangular fuzzy number can by denoted by (a, b, c).

(see Figure 1)

By the extension principle (Dubois and Prade, 1980), the fuzzy sum ⊕ and fuzzy subtraction ⊖ of any two triangular fuzzy numbers are also triangular fuzzy numbers; but the multiplication ⊗ of any two triangular fuzzy numbers is only an approximate triangular fuzzy number. Given any two positive triangular fuzzy numbers, $\widetilde{A} = (a_1, a_2, a_3), \widetilde{B} = (b_1, b, b_3)$ and a positive real number r, some main operations of fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ can be expressed as follows:

$$\begin{aligned} &\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \\ &\widetilde{A} \otimes \widetilde{B} \cong (a_1 b_1, a_2 b_2, a_3 b_3), \\ &-\widetilde{A} = (-a_3, -a_2, -a_1), \\ &\breve{A} \otimes r = (a_1 r, a_2 r, a_3 r), \\ &\widetilde{A}^{-1} = \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right) \qquad (1) \end{aligned}$$

Fuzzy decision: The fuzzy set of alternatives resulting from the intersection of the fuzzy constraints and fuzzy objective functions (Bellman & Zadeh, 1970). A fuzzy decision is defined in an analogy to non-fuzzy environments "as the selection of activities which simultaneously satisfy objective functions and constraints". Fuzzy objective function is characterized by its membership functions. In fuzzy set theory the intersection of sets normally corresponds to the logical "and". The "decision" in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy objective functions. The relationship between constraints and objective functions in a fuzzy environment is fully symmetric (Zimmerman, 1978).

Fuzzy Multi-Objective Problem

Relation between MODM, FMODM, FGP

A general linear multiple criteria decision making model can be presented as:

Find a vector **x** written in the transformed form $x^T = [x_1, x_2, \ldots, x_n]$

Which maximizes a vector of objective functions $(f_1, f_2, \ldots, f_n)$ where

$$f_i = \sum_j c_{ij} x_j \tag{2}$$

In other words:

$$\max f_i = \sum_{j=1}^{n} c_{ij} x_j \qquad , \forall\, i = 1,2, \ldots, m \tag{3}$$

with a system of constraints $G(x)$ defined as:

$$\sum_{j=1}^{n} a_{lj} x_j \le b_l \qquad l = 1,2, \ldots, L$$

$$x_j \ge 0 \qquad j = 1,2, \ldots, n \tag{4}$$

Where c_{ij}, a_{lj} and b_i are crisp (non-fuzzy) values. This problem has been studied and solved by many authors. Zimmermann has solved this problem by using the fuzzy linear programming (Zimmermann, 1978). He formulated the fuzzy linear program by separating every objective function f_i, its maximum f_i^+ and minimum f_i^- value by solving:

$$f_i^+ = \max\ f_i \qquad \forall i \tag{5}$$

S.t.

$$x \in G(x) \tag{6}$$

After finding the optimum solution X_j^* and maximum value f_i^+ for each objective f_i we can calculate their corresponding minimum value f_i^- as below:

$$f_i^- = \min_j \{f_i(X_j^*)\} \qquad \forall i \tag{7}$$

Since for every objective function f_i, its value changes linearly from f_i^- to f_i^+ it may be considered as a fuzzy number with the membership function $\mu_{(f_i)}$ as shown in Figure 1.

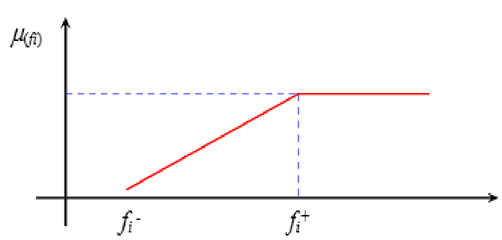

Figure 1. Objective function as a fuzzy number

So membership function of each decision x in optimum solutions set for objective function i can be formulated as below:

$$\mu_{f_i} = \begin{cases} 0 & f_i(x_j) < f_i^- \\ \frac{f_i(x)-f_i^-}{f_i^+-f_i^-} & f_i^- \le f_i(x) < f_i^+ \\ 1 & f_i(x_j) \ge f_i^+ \end{cases} \quad (8)$$

According to Bellman-Zadeh's principle of decision making in the fuzzy environment the grade of membership of a decision x denoted by α, designated by objectives f_i, is obtained by (Bellman & Zadeh, 1970):

$$\alpha = \min_i\{\mu_{f_i}(x)\} \quad (9)$$

According to this principle the optimal values of multi-criteria optimization problem is identical to the optimum value of the linear programming below:

$$\begin{aligned} &\text{Max } \alpha \\ &\alpha \le \mu_{f_i}(X) && \forall i \\ &\mu_{f_i}(X) = \frac{f_i(X) - f_i^-}{f_i^+ - f_i^-} && \forall i \\ &X \in G(X) \end{aligned} \quad (10)$$

Fuzzy Goal Programming Problem with Priorities

When we are solving the project selection with goal programming respects, membership functions are defined in another form. Membership function imputed by each criterion to each decision is allocated based on its similarity to aspiration level attached to corresponding criterion. in this way, Membership function is formulated so that maximum membership function value is assigned to Decisions which satisfy that criterion exactly identical to aspiration level and its value decreases linearly when it approaches to its maximum (f_i^+) and minimum (f_i^-) value (Chao-Fang Hu, et al. 2006). Figure 2 illustrates the behavior of this membership function.

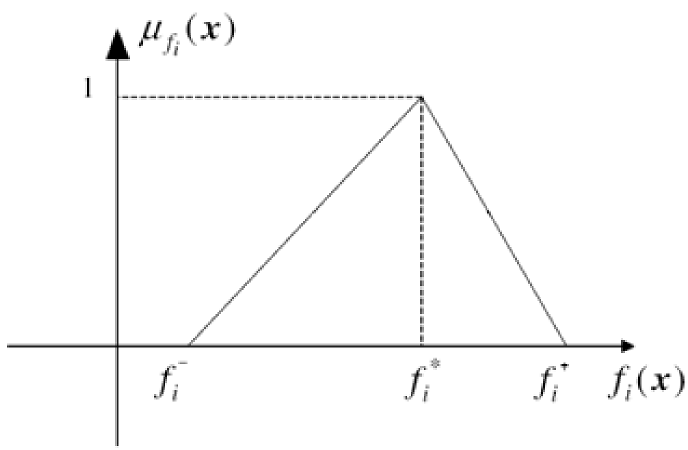

Figure 2. The fuzzy membership function

Where f_i^* is the goal value for i th objective given from DM. it is obvious that its values for each objective should lie between f_i^- and f_i^+. Membership function for each decision x can be formulated as:

$$\mu_{f_i}(x) = \begin{cases} 0 & f_i(x_j) < f_i^- \\ \frac{f_i(x)-f_i^-}{f_i^*-f_i^-} & f_i^- \le f_i(x) < f_i^* \\ 1 & f_i(x) = f_i^* \\ \frac{f_i^+-f_i(x)}{f_i^+-f_i^*} & f_i^* \le f_i(x) < f_i^+ \\ 0 & f_i(x) \ge f_i^+ \end{cases} \tag{11}$$

This type of membership function causes some difficulties as compared with that one defined in (8). Last membership function could be written in linear form easily, but this one could not be transformed into a linear equation exactly. But it can be transformed into a set of linear equations beside a binary variable required for each set. Membership function defined in (11) can be rewritten in the alternative form as below:

$$\mu_{f_i} = \frac{f_i(x) - f_i^-}{f_i^* - f_i^-} + \frac{f_i^+ - f_i(x)}{f_i^+ - f_i^*} \qquad \forall\, i$$

$$\frac{f_i(x)-f_i^-}{f_i^*-f_i^-} \le Mv_i \qquad \forall\, i \tag{12}$$

$$\frac{f_i^+ - f_i(x)}{f_i^+ - f_i^*} \le M(1 - v_i) \qquad \forall\, i$$

$$v_i \in \{0,1\} \qquad \forall\, i$$

It is worth to remind that by adding v_i variables to model, the computational efficiency of the proposed method doesn't decrease. Because usual problems arisen from real world don't generate huge number of binary variables. in other words in the worst case, real problems comprise at most less than 100 objective functions which required time to solve this size of binary linear programming problems is negligible.

In the GP problems, the DM usually has a preemptive priority requirement for achieving goals. That is, some goals have a higher priority for their achievement than the others under system constraints. This is different than of weight which is assigned to each criteria or objective by ANP. Because this kind of priorities results in "satisfying of a prioritized objective more than a disinterested one".

Conventional delaminating FGP method (Hannan, 1981, Tiwari et al. 1987, Chen 1994) is used when the DM has a priority order toward different goals. The m objectives are categorized into k levels according to the priority order, and the k sub-problems are solved in sequence, which results in low computation efficiency.

Suppose that objective $f_s(x)$ is more important than $f_t(x)$ ($s, t = 1,2,\dots,m$, and $s \neq t$), for all $x \in G(x)$, which is expressed as:

$$f_t(x) \prec f_s(x) \tag{13}$$

If two or more objectives have the same priority, we write as:

$$f_s(x) \sim f_t(x) \tag{14}$$

It is reasonable for us to hope that objectives with higher priorities will also have higher levels of satisfaction. If x^* is a solution to the FGP problem with multi-priority, then the conditions of priority (13) can be described as follows (Chen and Tasi 2001):

$$\mu_{f_t}(x^*) \le \mu_{f_s}(x^*) \tag{15}$$

In paper of Chen and Tasi (2001), the FGP with multiple priorities is modeled as follows:

$$\begin{cases} \max \quad \sum_{i=1}^{m} \mu_{f_i}(x) \\ s.t. \quad x \in F^{\alpha}(x) \cap G(x), \\ \quad \mu_{f_t}(x) \le \mu_{f_s}(x), \\ s,t = 1,2,\dots,m, \quad s \neq t. \end{cases} \tag{16}$$

by using definitions for membership functions $\mu_{f_i}(x)$ developed in this paper, equations (12) and also interdependency effects, we can write the final model as:

$$\max \quad \gamma\alpha + (1-\gamma)\sum w_i \mu_{f_i}(x)$$

$$s.t.$$

$$\alpha \le \mu_{f_i}(x) \qquad \forall\, i$$

$$\mu_{f_i} = \frac{f_i(x) - f_i^-}{f_i^* - f_i^-} + \frac{f_i^+ - f_i(x)}{f_i^+ - f_i^*} \qquad \forall\, i$$

$$\frac{f_i(x) - f_i^-}{f_i^* - f_i^-} \le M v_i \qquad \forall\, i \tag{17}$$

$$\frac{f_i^+ - f_i(x)}{f_i^+ - f_i^*} \le M(1 - v_i) \qquad \forall\, i$$

$$f_t(x) \le f_s(x) \qquad \forall\, s,t \mid \text{objective s is prioritized objective t}$$

$$x \in G(x)$$

$$v_i \in \{0,1\} \qquad \forall\, i$$

Where w_i's are weights associated to each objective considering interdependencies, and γ is a balancing parameter determined by user which is between 0 and 1. When it takes the value of 1, the model tries to maximize the minimum level of satisfaction to all objectives. Instead of this situation, when it takes the value of 0, the model tries to maximize the weighted summation of membership functions.

Numerical Example

Step1: The importance weights of various attributes and ratings of qualitative attributes are considered as linguistic variables. These linguistic variables can be expressed in positive triangular fuzzy numbers in Table1.

Table 1. Linguistic variables and their relative fuzzy numbers

Linguistic variable	Fuzzy number
Very Low (VL)	(0,0,3)
Low (L)	(0,3,5)
Medium (M)	(3,5,7)
High (H)	(5,7,10)
Very High (VH)	(7,10,10)

The linguistic variables and fuzzy weights related to the importance of criteria, assuming that there is no interdependency among them are given in Table2.

Table 2. Linguistic variables and fuzzy weights of criteria

criteria	Linguistic variable	Fuzzy number	Normalized weights
OL	H	(5,7,10)	(0.167,0.318,0.667)
AC	M	(3,5,7)	(0.1,0.227,0.467)
IC	VH	(7,10,10)	(0.233,0.455,0.667)
E	VL	(0,0,3)	(0,0,0.2)

Step2: Next, by assuming that there is no interdependence among projects, they are compared with respect to each criterion by linguistic variables, as shown in Table3 and relative fuzzy numbers are given in Table4. The second row of data in Table4 includes fuzzy numbers assigned to the degree of importance for each criterion, and the data of the third row are normalized weights for each criterion.

Table 3. Comparison of projects by four criteria

	OL	AC	IC	E
p1	H	M	L	M
p2	M	H	M	H
p3	VH	H	L	VH
p4	M	L	H	H
p5	L	H	H	H
p6	L	M	M	H

Table 4. Fuzzy numbers related to comparison of projects by four criteria

	OL	AC	IC	E
p_1	(5.000,7.000,10.000)	(3.000,5.000,7.000)	(0.000,3.000,5.000)	(3.000,5.000,7.000)
p_2	(3.000,5.000,7.000)	(5.000,7.000,10.000)	(3.000,5.000,7.000)	(5.000,7.000,10.000)
p_3	(7.000,10.000,10.000)	(5.000,7.000,10.000)	(0.000,3.000,5.000)	(7.000,10.00,10.000)
p_4	(3.000,5.000,7.000)	(0.000,3.000,5.000)	(5.000,7.000,10.000)	(5.000,7.000,10.000)
p_5	(0.000,3.000,5.000)	(5.000,7.000,10.000)	(5.000,7.000,10.000)	(5.000,7.000,10.000)
p_6	(0.000,3.000,5.000)	(3.000,5.000,7.000)	(3.000,5.000,7.000)	(5.000,7.000,10.000)
SUM	(18.000,33.000,44.000)	(21.000,34.000,49.000)	(16.000,30.000,44.000)	(30.000,43.000,57.000)
p_1	(0.114,0.212,0.556)	(0.061,0.147,0.333)	(0.000,0.100,0.313)	(0.053,0.116,0.233)
p_2	(0.068,0.152,0.389)	(0.102,0.206,0.476)	(0.068,0.167,0.438)	(0.088,0.163,0.333)
p_3	(0.159,0.303,0.556)	(0.102,0.206,0.476)	(0.000,0.100,0.313)	(0.123,0.233,0.333)
p_4	(0.068,0.152,0.389)	(0.000,0.088,0.238)	(0.114,0.233,0.625)	(0.088,0.163,0.333)
p_5	(0.000,0.091,0.278)	(0.102,0.206,0.476)	(0.114,0.233,0.625)	(0.088,0.163,0.333)
p_6	(0.000,0.091,0.278)	(0.061,0.147,0.333)	(0.068,0.167,0.438)	(0.088,0.163,0.333)
	W21	W22	W23	W24

Step3: In this step, we consider the interdependency among the criteria. One criteria's degree of relative impact for the other criteria is expressed as a fuzzy number which is expressed in interdependent fuzzy weight matrix of criteria as Table 5.

Table 5. Interdependency between criteria

	OL	AC	IC	E

OL	(0.9,1.0,1.0)	(0.1,0.2,0.3)	(0.0,0.0,0.1)	(0.0,0.0,0.1)
AC	(0.0,0.0,0.1)	(0.4,0.5,0.6)	(0.0,0.0,0.1)	(0.0,0.1,0.2)
IC	(0.0,0.0,0.1)	(0.2,0.3,0.4)	(0.9,1.0,1.0)	(0.3,0.4,0.5)
E	(0.0,0.0,0.1)	(0.0,0.0,0.1)	(0.0,0.0,0.1)	(0.4,0.5,0.6)

Step4: Next, we dealt with interdependency among the alternatives with respect to each criterion. The data are shown in Tables 6 to 9.

Step5: we now obtain the interdependence priorities of the criteria (OL, AC, IC, and E) as follows:

$$W_c = W_3 * W_1 = \begin{bmatrix} (0.160\ ,\ 0.364\ ,\ 0.893) \\ (0.040\ ,\ 0.114\ ,\ 0.453) \\ (0.230\ ,\ 0.523\ ,\ 1.020) \\ (0.000\ ,\ 0.000\ ,\ 0.030) \end{bmatrix}$$

Step6: The priorities of the projects W_p with respect to each criteria are given as follows:

$$W_{p_1} = W_{4_1} * W_{2_1} = \begin{bmatrix} (0.008\ ,\ 0.038\ ,\ 0.225) \\ (0.007\ ,\ 0.058\ ,\ 0.380) \\ (0.022\ ,\ 0.119\ ,\ 0.628) \\ (0.027\ ,\ 0.164\ ,\ 0.970) \\ (0.059\ ,\ 0.281\ ,\ 1.469) \\ (0.051\ ,\ 0.342\ ,\ 2.012) \end{bmatrix} \qquad W_{p_2} = W_{4_2} * W_{2_2} = \begin{bmatrix} (0.008\ ,\ 0.042\ ,\ 0.228) \\ (0.018\ ,\ 0.082\ ,\ 0.428) \\ (0.024\ ,\ 0.123\ ,\ 0.707) \\ (0.010\ ,\ 0.131\ ,\ 0.847) \\ (0.048\ ,\ 0.236\ ,\ 1.314) \\ (0.048\ ,\ 0.387\ ,\ 2.087) \end{bmatrix}$$

$$W_{p_3} = W_{4_3} * W_{2_3} = \begin{bmatrix} (0.004\ ,\ 0.043\ ,\ 0.368) \\ (0.008\ ,\ 0.057\ ,\ 0.444) \\ (0.003\ ,\ 0.072\ ,\ 0.679) \\ (0.008\ ,\ 0.141\ ,\ 1.202) \\ (0.032\ ,\ 0.263\ ,\ 2.314) \\ (0.073\ ,\ 0.425\ ,\ 3.254) \end{bmatrix} \qquad W_{p_4} = W_{4_4} * W_{2_4} = \begin{bmatrix} (0.006\ ,\ 0.025\ ,\ 0.106) \\ (0.019\ ,\ 0.075\ ,\ 0.349) \\ (0.016\ ,\ 0.089\ ,\ 0.354) \\ (0.042\ ,\ 0.171\ ,\ 0.676) \\ (0.055\ ,\ 0.233\ ,\ 0.938) \\ (0.101\ ,\ 0.407\ ,\ 1.785) \end{bmatrix}$$

Table 6. Interdependency among the alternatives with respect to criterion OL

OL	p1	p2	p3	p4	p5	p6
p1	(0.061,0.135,0.304)	(0.005,0.013,0.070)	(0.004,0.009,0.025)	(0.005, 0.009 ,0.021)	(0.006,0.008,0.017)	(0.012,0.025,0.056)
p2	(0.000,0.081,0.217)	(0.081,0.197,0.493)	(0.006,0.015,0.075)	(0.007, 0.013 ,0.036)	(0.006,0.011,0.024)	(0.017,0.034,0.093)
p3	(0.061,0.135,0.304)	(0.000,0.118,0.352)	(0.090,0.222,0.525)	(0.005, 0.009 ,0.021)	(0.006,0.011,0.024)	(0.012,0.025,0.056)
p4	(0.102,0.189,0.435)	(0.081,0.197,0.493)	(0.000,0.133,0.375)	(0.138, 0.323 ,0.749)	(0.008,0.016,0.039)	(0.017,0.034,0.093)
p5	(0.143,0.270,0.435)	(0.135,0.276,0.704)	(0.150,0.311,0.749)	(0.138, 0.323 ,0.749)	(0.168,0.397,0.829)	(0.012,0.025,0.056)

p6	(0.102,0.189,0.435)	(0.081,0.197,0.493)	(0.150,0.311,0.749)	(0.138, 0.323 ,0.749)	(0.280,0.556,1.184)	(0.363,0.858,1.952)

Table 7. Interdependency among the alternatives with respect to criterion AC

AC	p1	p2	p3	p4	p5	p6
p1	(0.102, 0.206, 0.476)	(0.003, 0.007, 0.017)	(0.005, 0.011, 0.036)	(0.009, 0.021, 0.059)	(0.005, 0.011, 0.023)	(0.012, 0.024, 0.055)
p2	(0.061, 0.147, 0.333)	(0.121, 0.240, 0.522)	(0.005, 0.011, 0.036)	(0.006, 0.013, 0.039)	(0.005, 0.011, 0.023)	(0.017, 0.034, 0.092)
p3	(0.061, 0.147, 0.333)	(0.073, 0.171, 0.366)	(0.113, 0.272, 0.754)	(0.009, 0.021, 0.059)	(0.008, 0.016, 0.039)	(0.017, 0.034, 0.092)
p4	(0.000, 0.088, 0.238)	(0.073, 0.171, 0.366)	(0.000, 0.163, 0.538)	(0.214, 0.441, 1.171)	(0.011, 0.026, 0.059)	(0.017, 0.034, 0.092)
p5	(0.102, 0.206, 0.476)	(0.121, 0.240, 0.522)	(0.113, 0.272, 0.754)	(0.000, 0.189, 0.585)	(0.165, 0.390, 0.819)	(0.012, 0.024, 0.055)
p6	(0.102, 0.206, 0.476)	(0.073, 0.171, 0.366)	(0.113, 0.272, 0.754)	(0.129, 0.315, 0.819)	(0.274, 0.546, 1.171)	(0.357, 0.850, 1.929)

Table 8. Interdependency among the alternatives with respect to criterion IC

IC	p1	p2	p3	p4	p5	p6
p1	(0.114,0.233, 0.625)	(0.005,0.013, 0.035)	(0.006,0.012, 0.053)	(0.004,0.009, 0.024)	(0.015,0.038, 0.139)	(0.016,0.033,0.089)
p2	(0.000,0.100, 0.313)	(0.082,0.197, 0.493)	(0.008,0.020, 0.079)	(0.006,0.013, 0.040)	(0.011,0.023, 0.093)	(0.011,0.024,0.054)
p3	(0.068,0.167, 0.438)	(0.000,0.118, 0.352)	(0.000,0.181, 0.788)	(0.006,0.013, 0.040)	(0.011,0.023, 0.093)	(0.023,0.055,0.134)
p4	(0.114,0.233, 0.625)	(0.082,0.197, 0.493)	(0.121,0.302, 1.104)	(0.000,0.193, 0.596)	(0.007,0.016, 0.056)	(0.016,0.033,0.089)
p5	(0.000,0.100, 0.313)	(0.082,0.197, 0.493)	(0.121,0.302, 1.104)	(0.219,0.450, 1.193)	(0.000,0.338, 1.394)	(0.016,0.033,0.089)
p6	(0.068,0.167, 0.438)	(0.137,0.276, 0.704)	(0.000,0.181, 0.788)	(0.131,0.322, 0.835)	(0.224,0.563, 1.952)	(0.345,0.823,1.877)

Table 9. Interdependency among the alternatives with respect to criterion E

E	p1	p2	p3	p4	p5	p6
p1	(0.056, 0.132 ,0.269)	(0.003, 0.005 ,0.012)	(0.004, 0.008 ,0.020)	(0.004, 0.008 ,0.018)	(0.006,0.011,0.024)	(0.012,0.025,0.056)
p2	(0.093, 0.184 ,0.385)	(0.128, 0.258 ,0.621)	(0.006, 0.014 ,0.031)	(0.006, 0.011 ,0.029)	(0.008,0.016,0.039)	(0.012,0.025,0.056)
p3	(0.056, 0.132 ,0.269)	(0.000, 0.111 ,0.311)	(0.086, 0.204 ,0.428)	(0.004, 0.008 ,0.018)	(0.006,0.011,0.024)	(0.018,0.035,0.094)
p4	(0.093, 0.184 ,0.385)	(0.077, 0.184 ,0.435)	(0.144, 0.285 ,0.612)	(0.121, 0.286 ,0.617)	(0.008,0.016,0.039)	(0.012,0.025,0.056)

p5	(0.093, 0.184 ,0.385)	(0.077, 0.184 ,0.435)	(0.144, 0.285 ,0.612)	(0.121, 0.286 ,0.617)	(0.166,0.394,0.825)	(0.012,0.025,0.056)
p6	(0.093,0.184, 0.385)	(0.128,0.258, 0.621)	(0.086,0.204, 0.428)	(0.202,0.400, 0.882)	(0.277,0.552,1.178)	(0.369,0.866,1.976)

Step7: Finally, we define the matrix W_p as $W_p= (W_{p1}, W_{p2}, W_{p3}, W_{p4})$ and the overall priorities for the projects are calculated by multiplying W_p by W_c.

$$W = W_p * W_c = \begin{bmatrix} (0.003 \ , 0.041 \ , 0.712) \\ (0.004 \ , 0.060 \ , 1.091) \\ (0.005 \ , 0.095 \ , 1.680) \\ (0.006 \ , 0.148 \ , 2.680) \\ (0.019 \ , 0.266 \ , 4.550) \\ (0.028 \ , 0.390 \ , 6.599) \end{bmatrix}$$

To obtain the final results of the ANP phase a ranking method is needed. In this paper the graded mean integration representation method proposed by Chen and Hsieh (Saaty TL , 1980) is used to rank the final ratings of alternatives.

Let $A_i = (a_i, b_i, c_i)$, $i = 1, 2, ..., n$, be n triangular fuzzy numbers. By the graded mean integration representation method, the graded mean integration representation $P(A_i)$ of A_i is

$$P(A_i) = \frac{a_i + 4b_i + c_i}{6} \tag{18}$$

Suppose $P(A_i)$ and $P(A_j)$ are the graded mean integration representations of the triangular fuzzy numbers A_i and A_j respectively.

Define that

$A_i > A_j \Leftrightarrow P(A_i) > P(A_j)$,

$A_i < A_j \Leftrightarrow P(A_i) < P(A_j)$,

$A_i = A_j \Leftrightarrow P(A_i) = P(A_j)$.

According to this method of ranking, the final ranks of alternatives under criteria are as follows:

	p_1	p_2	p_3	p_4	p_5	p_6
Final Weights	0.146	0.223	0.344	0.546	0.939	1.365

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