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Retraction of null helix in Minkowski 3-space

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ABSTRACT: Our aim in the present article is to introduce and study new types of retraction of null helix in Minkowski 3-space.Types of the deformation retracts of the null helix in Minkowski 3-space is presented. The relations between the retraction and the deformation retract of null helix are deduced. Types of minimal retraction of null helix in Minkowski 3-space are also presented. New types of homotopy maps are described.
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Introduction And Definitions

As is well known, the theory of retraction is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry (El-Ahmady, 2013 a, b, c, d, e, f), (El-Ahmady, 2011 b, c), (El-Ahmady, 2007 b), (El-Ahmady, 2006),

An n-dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n.If $h:U \to U'$ is a homeomorphism of $U \subseteq M$ onto $U' \subseteq R^n$,then h is called a chart of M and U is the associated chart domain. A collection (h_{α, U_α}) is said to be an atlas for M if $\cup_{\alpha \in A} U_\alpha = M$. Given two charts h_α, h_β such that $U_{\alpha\beta} = U_\alpha \cap U_\beta \neq \emptyset$,the transformation chart $h_\beta \circ h_\alpha^{-1}$ between open sets of R^n is defined ,and if all of these charts transformation are c^∞-mappings, then the manifolds under consideration is a c^∞ - manifolds .A differentiable structure on M is a differentiable atlas and a differentiable manifolds is a topological manifolds with a differentiable structure (Arkowitz, 2011), (Reid, 2005), (Shick, 2007) and (Strom, 2011).

A subset A of a topological space X is called a retract of X if there exists a continuous map $r: X \to A$ such that $(a) = a \; \forall a \in A$, where A is closed and X is open (El-Ahmady, 2012a, b, c).
This can be restated as follows. If $i: A \to X$ is the inclusion map, then $r: X \to A$ is a map such that $r \circ i = Id_A$.
Another simple-but extremely useful-idea is that of a retract. If $A, X \subset M$, then A is a retract of X if there is a commutative diagram (El-Ahmady, 2011a).

$$\begin{tikzcd} A \arrow[r, "i"'] \arrow[rr, bend left, "Id_A"] & X \arrow[r, "r"'] & A \end{tikzcd}$$

If $f : A \to B$ and $g : X \to Y$, then f is a retract of g if there is a commutative diagram (Naber, 2011).

$$\begin{tikzcd}
A \arrow[r, "i"] \arrow[d, "f"'] \arrow[rr, bend left, "Id_A"] & X \arrow[r, "r"] \arrow[d, "g"] & A \arrow[d, "f"] \\
B \arrow[r, "j"'] \arrow[rr, bend right, "Id_B"'] & Y \arrow[r, "s"'] & B
\end{tikzcd}$$

that is, $r \circ i = Id_A$, $s \circ j = Id_B$, $s \circ j \circ f = f \circ r \circ i$, $g \circ i = j \circ f$,and $f \circ r = s \circ g$.

A subset A of a topological space X is a deformation retract of X if there exists a retraction $r{:}\,X \to A$ and a homotopy $\emptyset{:}\,X \times \mathrm{I} \to X$such that (Naber, 2011)

$\emptyset\ (x, 0) = x$

$\emptyset\ (x, 1) = r(x)$

$\emptyset(a, t) = a,\ a \in A, t = [0,1].\ x \in X$

Also, suppose X is a topological space, A is a subspace of X, and $r{:}\,X \to A$ is a retraction. We say that r is a deformation retract if $i_A \circ r$ is homotopic to the identity map of X, where $i_A{:}\,A \to X$ is the inclusion map. If there exists a deformation retract from X to A, then, A is said to be a deformation retract of X. Because $i_A \circ r \simeq Id_X$ and $r \circ i_A = Id_A$. It follows that both i_A and r are homotopy equivalence (Strom, 2011).

Let $\overline{M}^{n+1} \cong M_{012\ldots h}$ be the Minkowski (n+1)-space ,that is, $\overline{M}^{n+1}$ is the real vector space $\widetilde{R}^{n+1}$ endowed with the standard flat metric $ds^{\tilde{\tilde{2}}} = \sum_{i=1}^{n-1} dx_i^{\tilde{\tilde{2}}} - dx_n^{\tilde{\tilde{2}}}$. Also , $ds^{\tilde{\tilde{2}}} > 0$, $ds^{\tilde{\tilde{2}}} = 0$ and $ds^{\tilde{\tilde{2}}} < 0$ correspond to space –like ,null and time –like geodesics (James, 2003).

Helix is one of the most fascinating curves in Science and Nature .From the view of differential geometry ,a helix is a geometry curve with non –vanishing constant curvature (or first curvature of the curve and denoted by k_1) and non-vanishing constant torsion (or second curvature of the curve and denoted by k_2).

Indeed a helix is a special case of the general helix .A curve of constant slope or general helix in Euclidean 3-space E^3 ,is defined by the property that the tangent makes a constant angle with a fixed straight line(Yaliniz, 2007), It is well known that to each unit speed curve $\alpha : \mathrm{I} \subset R \to R^3$ in the Euclidean space R^3 whose successive derivatives $\alpha'(s)$, $\alpha''(s)$ and $\alpha'''(s)$are linearly independent vectors ,one can associate three mutually orthogonal unit vector field T,N and B called respectively the tangent ,the principle normal and the binormal vector field(Ceylan, 2007). At each point $\alpha(s)$ of curve α ,the planes spanned by {T,N } ,{T,B } and {N,B } are known respectively as the osculating plane ,the rectifying plane and the normal plane(Kocayigit, 2007). The curves $\alpha : \mathrm{I} \subset R \to R^3$ for which the position vector α always lie in their rectifying plane ,are simplicity called rectifying curves .Similarly ,the curves for which the position vector α always lie in their osculating plane , are for simplicity called osculating curves. Moreover , the curves for which the position vector α always lie in their normal plane , are for simplicity called normal curves(Ilarslan, 2004).

The Minkowski 3-space E_1^3 is the Euclidean 3-space E^3 provided with the standard flat metric given by $g = -dx_1^2 + dx_2^2 + dx_3^2$,

Where (x_1, x_2, x_3) is a rectangular coordinate system of E_1^3 .Since g is an indefinite metric ,recall that a vector $v \in E_1^3$ can have one of three Lorentzian causal characters ,it can be space like if $g(v, v) > 0 or\ v = 0$,time like if $g(v, v) < 0$ and light like if $g(v, v) = 0\ and\ v \neq 0$ .Similarly, an arbitrary curve $\alpha = \alpha(s)$ in E_1^3 can locally be space like ,time like or light like ,if all of its velocity vectors $\alpha'(s)$ are respectively , space like ,time like or light like respectively (Onder, 2006).

Minkowski space is originally from the relativity in physics .In fact ,a time like curve corresponds to the path of an observer moving at less than the speed of light , a light like curves correspond to moving at the speed of light and a space like curves moving faster than light (El-Ahmady, 2013 a, b, c, d).

Consider the moving Frenet frame {T,N,B} along the curve $\alpha(s)$ in E_1^3.For an arbitrary curve $\alpha(s)$ in the space E_1^3 ,the following Frenet formula are given in (El-Ahmady, 2013 a, b, c, d).

A curve in Lorentzian space L^n is a smooth map $\alpha : I \to L^n$ where $I$is the open interval in the real line R .The interval Ihas a coordinate system consisting of the identity map u of I .The velocity of α at $t \in I$ is $\alpha' = \frac{d\alpha(u)}{du}\Big|_t$ .A curve α is said to be regular if $\alpha'(t)$ does not vanish for all $t\ in\ I$. $\alpha \in L^n$ is space like if its velocity vectors $\alpha'$are space like for all $t \in I$,similarly for time like and null. If α is a null curve ,we can reparametrize it such that $< \alpha'(t)\ .\alpha'$ (t)>$= 0$ and $\alpha'(t) \neq 0$ (Naber, 2011).

If α is null curve ,then the Frent formula read

$$\begin{pmatrix} T' \\ N' \\ B' \end{pmatrix}=\begin{pmatrix} O & K_1 & O \\ K_2 & 0 & -K_1 \\ O & -K_2 & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix}$$

Where $g(T,T) = g(B,B) = 0, g(N,N) = 1, g(T,N) = g(N,B) =0$, $g(T,B) =1$.In this case , K_1can take only two values $K_1 = 0$ when α is straight line or $K_1 = 1$ in all other(El-Ahmady, 2013 a, b, c, d).

The importance of the study of null curve and its presence in the physical theories is clear from the fact that the classical relativistic string is a surface or world –sheet in Minkowski space which satisfies the Lorentzian analogue of minimal surface equation .The string equations simplify to the wave equation and a couple of extra simple equations, and by solving the 2-dimentional wave equations it turns out that string are equivalent to pairs of null curves ,or a single null curve in the case of an open string . In this paper we will introduce a new type of retractions of null curve in Minkowski space from view point of the variation of the density on chaotic spheres in chaotic space-like Minkowski space time and retraction of chaotic black hole, as presented by (El-Ahmady, A .E. 2007a)and (El-Ahmady, 2011c).

The Main Results

Let $\alpha(s)$ be a null helix .Then we can write its position vector as follows:

$$\alpha(S) = \lambda(s)T(s) + \mu(s)N(s) + \upsilon(s)B(s) \quad (1)$$

Differentiating equation (1) with respect to s and using Frenet formula we have

$$\lambda'(s) + \mu(s)K_2 = 1\ , \lambda(s)k_1 + \mu'(s) - \nu(s)K_2 = 0, -\mu((s)K_1 + \nu'(s)=0 \quad (2)$$

From equation (2) we have

$$\mu'' - \mu(s)(K_2^2 + K_1K_2\ \ ) + K_2 = 0 \quad (3)$$

Since $\alpha(s)$ be a null helix, $K_1 = 1$ .Then we can write equation (3) as follows:

$$\mu'' - \mu(s)(K_2^2 + K_2\ \ ) + K_2 = 0 \quad (4)$$

Now we are going to discuss the following case

If $K_2 = -1$.The solution of equation (4) is

$$\mu(s) = \frac{S^2}{2} + C_1\mathrm{S} + C_2 \quad (5)$$

Where $c_{1,}c_2 \epsilon R$ .From equation (2) we have $\lambda'(s) = 1$- $\mu(s)K_2$.By using (5) we find the solution of this equation as follows:

$$\lambda(s) = S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S \quad (6)$$

Also from (2) we have $\nu'(s) = \mu((s)K_1$ .By using (5) we get

$$\nu(s) = \frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3 \quad (7)$$

Thus we find the position vector as

$$\alpha_1(s) = (S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^2}{2} + C_1\mathrm{S} + C_2)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s) \quad (8)$$

Now we are going to discuss the retraction of the position vector $\alpha_1(s)$ as follows

Let r_i :$\{\alpha_1(s) - \beta\} \to \{\alpha_1(s) - \beta\}^*$,where $\{\alpha_1(s) - \beta\}$ be the open helix and $\{\alpha_1(s) - \beta\}^*$ be the retraction of the position vector $\alpha_1(s)$.

Now we discuss the following cases:

If C_1=0,we have the following retraction defined as:

$$r_1(\alpha_1(s))=\left\{(S + \frac{S^3}{6} + C_2S)T(s) + (\frac{S^2}{2} + C_2)N(s) + (\frac{S^3}{6} + C_2S + C_3)B(s)\right\}$$

Also ,if C_2=0 we obtain the following retraction defined by:

$$r_2(\alpha_1(s))=\left\{(S + \frac{S^3}{6} - \frac{C_1}{2}S^2)T(s) + (\frac{S^2}{2} + C_1\mathrm{S})N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_3)B(s)\right\}$$

If $C_3 = 0$, we present the following retraction given by $r_3(\alpha_1(s)) = \Big\{ (S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^2}{2} + C_1S + C_2)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S)B(s) \Big\}$

If $S = 0$ we have the following retraction defined as:

$r_4(\alpha_1(s))=\{ C_2)N(s) + C_3)B(s) \}$

If $T(s) = 0$, we present the following retraction given by:

$r_5(\alpha_1(s))=\Big\{(\frac{S^2}{2} + C_1S + C_2)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s) \Big\}$

Also ,if $N(s)$ =0 we obtain the following retraction defined by:

$r_6(\alpha_1(s))=\Big\{ (S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s) \Big\}$

Now ,if $B(s) = 0$ we present the following retraction given by:

$r_7(\alpha_1(s))=\Big\{ (S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^2}{2} + C_1S + C_2)N(s) \Big\}$

Also ,if $T(s) = N(s)$ =0 , we have the following retraction defined as:

$r_8(\alpha_1(s))=\Big\{ (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s) \Big\}$

Now ,if $T(s) = B(s) = 0$ we present the following retraction given by:

$r_9(\alpha_1(s))=\Big\{ (\frac{S^2}{2} + C_1S + C_2)N(s) \Big\}$

Also ,if $N(s)= B(s) = 0$, we have the following retraction defined as:

$r_{10}(\alpha_1(s))=\Big\{ (S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) \Big\}$

Moreover ,if $T(s) = N(s) = B(s)$ =0 , we have the following retraction defined as:

$r_{11}(\alpha_1(s))=\{ 0,0,0\}$.

In this position ,we present some cases of deformation retract of a null helix in Minkowski 3-space .The deformation retract of a null helix is

φ: $\{\alpha_1(s) - \beta\} \times \mathrm{I} \to \{\alpha_1(s) - \beta\}$,

where $\{\alpha_1(s) - \beta\}$ be the open helix of the position vector $\alpha_1(s)$ and I is the closed interval [0,1],be present as

$$\varphi(x,h): \Big\{ \Big(\Big(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\Big)T(s) + \Big(\frac{S^2}{2} + C_1S + C_2\Big)N(s) + \Big(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\Big)B(s)\Big) - \beta \Big\} \times \mathrm{I}$$
$$\to \Big\{\Big(\Big(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\Big)T(s) + \Big(\frac{S^2}{2} + C_1S + C_2\Big)N(s) + \Big(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\Big)B(s)\Big) - \beta \Big\}.$$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_1(\alpha_1(s))$ is

$$\varphi(x,h) = (1-h)\Big\{\Big(\Big(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\Big)T(s) + \Big(\frac{S^2}{2} + C_1S + C_2\Big)N(s) + \Big(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\Big)B(s)\Big) - \beta \Big\} + h\Big\{ (S + \frac{S^3}{6} + C_2S)T(s) + (\frac{S^2}{2} + C_2)N(s) + (\frac{S^3}{6} + C_2S + C_3)B(s) \Big\}.$$

Where

$$\varphi(x,0) = \Big\{\Big(\Big(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\Big)T(s) + \Big(\frac{S^2}{2} + C_1S + C_2\Big)N(s) + \Big(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\Big)B(s)\Big) - \beta \Big\}$$

and

$$\varphi(x,1) = \Big\{(S + \frac{S^3}{6} + C_2S)T(s) + (\frac{S^2}{2} + C_2)N(s) + (\frac{S^3}{6} + C_2S + C_3)B(s) \Big\}.$$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_2(\alpha_1(s))$ is defined as

$$\varphi(x,h) = \cos\frac{\pi h}{2}\Big\{\Big(\Big(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\Big)T(s) + \Big(\frac{S^2}{2} + C_1S + C_2\Big)N(s) + \Big(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\Big)B(s)\Big) - \beta \Big\} + \sin\frac{\pi h}{2}\Big\{ (S + \frac{S^3}{6} - \frac{C_1}{2}S^2)T(s) + (\frac{S^2}{2} + C_1S)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_3)B(s) \Big\}.$$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_3(\alpha_1(s))$ is defined by

$\varphi(x,h) = \frac{1-h}{1+h}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \frac{2h}{1+h}\left\{(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^2}{2} + C_1S + C_2)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S)B(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_4(\alpha_1(s))$ is given by

$\varphi(x,h) = \cos\frac{\pi h}{2}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \sin\frac{\pi h}{2}\{C_2)N(s) + C_3)B(s)\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_5(\alpha_1(s))$ is defined as

$\varphi(x,h) = \frac{1-h}{1+h}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \frac{2h}{1+h}\left\{(\frac{S^2}{2} + C_1S + C_2)N(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_6(\alpha_1(s))$ is

$\varphi(x,h) = (1-h)\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + h\left\{(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_7(\alpha_1(s))$ is defined as

$\varphi(x,h) = e^h(1-h)\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \frac{h}{2}(2h + \frac{1}{2})\left\{(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s) + (\frac{S^2}{2} + C_1S + C_2)N(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_8(\alpha_1(s))$ is defined by

$\varphi(x,h) = \frac{1-h}{1+h}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \frac{2h}{1+h}\left\{(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3)B(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_9(\alpha_1(s))$ is defined as

$\varphi(x,h) =$
$\ln e^{(1-h)}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \ln e^h\left\{(\frac{S^2}{2} + C_1S + C_2)N(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_{10}(\alpha_1(s))$ is defined as

$\varphi(x,h) =$
$e^h(1-h)\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \frac{h}{2}(2h + \frac{1}{2})\left\{(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S)T(s)\right\}.$

The deformation retract of the helix $\alpha_1(s)$ into the retraction $r_{11}(\alpha_1(s))$ is defined as

$\varphi(x,h) =$
$\ln e^{(1-h)}\left\{\left(\left(S + \frac{S^3}{6} - \frac{C_1}{2}S^2 + C_2S\right)T(s) + \left(\frac{S^2}{2} + C_1S + C_2\right)N(s) + \left(\frac{S^3}{6} + \frac{C_1}{2}S^2 + C_2S + C_3\right)B(s)\right) - \beta\right\} + \ln e^h\{0,0,0\}.$

Conclusion

Consider a curve in a space suppose that the curve is sufficiently smooth so that the Frent frame adapted to is defined the curvature k_1 and torsion $k_2$then provide a complete characterization of the curve .Helix is one of the most fascinating curves in Science and Nature ,a helix is a geometry curve with non – vanishing constant curvature k_1 and non-vanishing constant torsion k_2.A curve of constant slope or general helix in Euclidean 3-space E^3 ,is defined by the property that the tangent makes a constant angle with a fixed straight line .In the present article, we obtain and study types of retraction of the position vectors of a null helix in Minkowski 3-space.Also ,by using the position vectors of the curve and retraction of the position vectors ,we deduced types of the deformation retracts of the a null helix in Minkowski 3-space. The relations between the retraction and the deformation retracts of a null helix are obtained. Types of minimal retraction of a null helix in Minkowski 3-space are also presented. New types of homotopy map are obtained

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