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# Equations in Permutation and Combinations with Introduction of the Anekwe's Method of Swapping Factorials

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***Abstract:*** *Permutations and combinations is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them. A permutation involves an arrangement of objects in a definite order while A Combinations On the other hand is not interested in arranging but only in selecting r objects from given n objects. A combination is a selection of some or all of a number of different objects where the order of selection is not important. In this Manuscript we are concerned about solving equations of permutation and combination, on how to determine n number of items and how it can be selected or arranged in r number of ways and also to introduce Anekwe□s method of swapping factorials in the case of determining r number of items.*

***Keywords:*** *Solve, Equation, Substituting.*

## INTRODUCTION

The study of permutations and combinations is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them. A permutation is an arrangement of objects in a definite order.

Combinations On many occasions we are not interested in arranging but only in selecting r objects from given n objects. A combination is a selection of some or all of a number of different objects where the order of selection is immaterial.

There are some basic counting techniques which will be useful in determining the number of different ways of arranging or selecting objects. The two basic counting principles are

**Multiplication principle:** Suppose an event E can occur in m different ways and associated with each way of occurring of E, another event F can occur in n different ways, then the total number of occurrence of the two events in the given order is $m \times n$.

**Addition principle:** If an event E can occur in m ways and another event F can occur in n ways, and suppose that both can not occur together, then E or F can occur in $m + n$ ways.

**Remarks**

Use permutations if a problem calls for the number of arrangements of objects and different orders are to be counted.

Use combinations if a problem calls for the number of ways of selecting objects and the order of selection is not to be counted.[1]

In this Manuscript we are concerned about solving equations of permutation and combination, on how to determine n number of items and how it can be selected or arranged in r number of ways and also to introduce The Anekwe□s method of swapping factorials in the case of determining r number of items.

## Methodology

This section involves solving equations in permutation and combination and also to introduce Anekwe□s method of swapping factorials in the case of determining r number of items through worked examples.

**Example 2.1**

Solve the equation

$$\frac{np5}{nc3}=12$$

Solution

$$\frac{\left(\frac{n!}{n-5!}\right)}{\left(\frac{n!}{n-3!3!}\right)}=12 \quad (1)$$

$$\frac{n!}{n-5!}*\frac{n-3!3!}{n!}=\frac{n-3!3!}{n-5!}=12 \quad (2)$$

By Cross multiplying both sides of equation (2) we□ve

$$12(n-5)!=n-3!3! \quad (3)$$

$\Rightarrow$ we□ve

$$12(n-5)(n-4)(n-3)(n-2)$$
$$=(n-3)(n-2)(6) \quad (4)$$

$$12(n-5)(n-4)=6 \quad (5)$$

$$2\left[n^2-9n+20\right]=1 \quad (6)$$

$$2n^2-18n+40=1 \quad (7)$$

$$2n^2-18n+39=0 \quad (8)$$

Using the general formula we□ve

$$n=\frac{-(-18)\pm\sqrt{(-18)^2-4*2*39}}{2*2}=18\pm\frac{3*5}{4} \quad (9)$$

$\Rightarrow n=5$

Or $n=4$. (10)

But on substitution n=4 does not satisfy the equation $\therefore\ n=5$

[1] National Council of Educational Research and Training Permutations and Combinations http://ncert.nic.in/ncerts/l/keep207.pdf

**Alternatively**

$$\frac{np5}{nc3}=12$$
$$np5=12*nc3$$

From the rule of permutation

$$\mathrm{np_r}=n(n-1)(n-2).....(n-1+r)$$
$$\mathrm{nc_r}=\frac{n(n-1)(n-2).....(n-1+r)}{r!}$$

$$\mathrm{np_5}=12*nc_3$$
$$n(n-1)(n-2)(n-3)(n-4)$$
$$=12*\frac{n(n-1)(n-2)}{3!}$$
$$3!*n(n-1)(n-2)(n-3)(n-4)$$
$$=12*n(n-1)(n-2)$$
$$(n-3)(n-4)=2$$
$$n^2-7n+12=2$$
$$n^2-7n+10=0$$
$$n^2-5n-2n+10=0$$
$$n(n-5)-2(n-5)=0$$
$$(n-2)(n-5)=0$$
$$\mathrm{n=2\ or\ n=5}$$

**Example 2.2**

Solve the following

$$\frac{np3}{nc3}=6$$

**Solution**

$$\frac{np3}{nc3}=6 \qquad (1)$$

$$\boxed{\frac{\frac{n!}{(n-3)!}}{\frac{n!}{3!(n-3)!}} = 6} \quad (2)$$

$$\boxed{\frac{n!}{n-3!} * \frac{n-3!3!}{n!} = 6} \quad (3)$$

$$\boxed{\frac{n-3!3!}{n-3!} = 6} \quad (4)$$

$$\frac{n-3!3!}{n-3!} - 6 = 0 \quad (5)$$

$$\frac{n-3!3! - 6(n-3)!}{n-3!} = 0$$

$$(n-3)!3! - 6(n-3)! = 0$$

$$(n-3)!3! - 6(n-3)! = 0$$

$$6(n-3)! = 6(n-3)! \quad (6)$$

Since both the left hand side and the right hand side are equal we will set one of them to be equal to zero in order to get the corresponding values of n.

$\therefore$ From the equation above we□ve

$$6(n-3)! = 0 \quad (7)$$

$$(n-3)(n-2) = 0$$

$$n^2 - 5n + 6 = 0 \quad (8)$$

On solving the above equation we□ve

$n = 3$ Or $n = 2$

But on substitution n=2 does not satisfy the equation $\therefore \ n = 3$

**Example 2.3**

Given that $4c_r = 6c_{r+3}$ find the values of r.

<u>Solution</u>

$$4c_r = 6c_{r+3}$$

$$\frac{4!}{r!(4-r)!} = \frac{6!}{(r+3)!(6-(r+3))!}$$

$$\frac{4!}{r!(4-r)!} = \frac{6!}{(r+3)!(3-r)!}$$

$$4!(3-r)!(r+3)! = 6!(4-r)!r!$$

**Using Anekwe□s method of Swapping Factorials we□ve**

Multiply all through the equation above by $\frac{1}{1!}$

$$4(3-r)(r+3)=6(4-r)r$$

$\Rightarrow$ On simplifying the equation above we've

$$2r^2-24r+36=0$$

Using general formula to solve the above equation we've

$$r=\frac{-(-24)\pm\sqrt{24^2-4*2*36}}{2*2}=\frac{24\pm17}{4}$$

$\Rightarrow$ r=2 or r=10

**Applications of The Principles of Permutation and Combination**

The Principles of Permutation and Combination has a lot of applications amongst which we've its Applications in:

- The Telephone Numbering System (What is the practical real life use of permutation and combination, 2014; Uchenna Okwudili Anekwe, 2017)
- Computer architecture
- Databases and data mining.
- Patern Analysis
- Operations research.
- Homeland security e.t.c

**1. Communication networks, cryptography and network security**

Permutations are frequently used in communication networks and parallel and distributed systems (Munakata, 1998; Stroud, Dexter and Booth, 2013). Routing different permutations on a network for performance evaluation is a common problem in these fields. Many communication networks require secure transfer of information, which drives development in cryptography and network security (Kumar Verma and Trimbak Tamhankar, 1997; Valdes-Perez, 1999). This area has recently become particularly significant because of the increased use of internet information transfers. Associated problems include protecting the privacy of transactions and other confidential data transfers and preserving the network security from attacks by viruses and hackers. Encryption process involves manipulations of sequences of codes such as digits, characters, and words. Hence, they are closely related to combinatorics, possibly with intelligent encryption process. For example, one common type of encryption process is interchanging--i.e., permuting parts of a sequence (Siepel, 2003). Permutations of fast Fourier transforms are employed in speech encryption (ICECS 2000).

**2. Computer architecture**

Design of computer chips involves consideration of possible permutations of input to output pins. Fieldprogrammable interconnection chips provide user programmable interconnection for a desired permutation (Bhatia and Haralambides, 2000). Arrangement of logic gates is a basic element for computer architecture design (Yang and Wang, 2004).

**3. Computational molecular biology**

This field involves many types of combinatorial and sequencing problems such as atoms, molecules, DNAs, genes, and proteins (Combinatorial Pattern Matching, 1992-2005; Kapralski, 1993; Tanenbaum, 1999). Onedimensional sequencing problems are essentially permutation problems under certain constraints.

**4. Languages**

Both natural and computer languages are closely related to combinatorics (Kapralski, 1993). This is because the components of these languages, such as sentences, paragraphs, programs, and blocks, are arrangements of smaller elements, such as words, characters, and atoms. For example, a string searching algorithm may rely on combinatorics of words and characters. Direct applications of this can include word processing and databases.

Another important application area is performance analysis of these string searching algorithms. The study of computability--what we can compute and how it is accomplished--draws heavily on combinatorics.

**5. Pattern analysis**

In a broad sense, all the above-mentioned areas can be viewed as special cases of pattern analysis.

**6. Molecular biology**

Molecular biology, for example, studies patterns of atoms, molecules, and DNAs whereas languages treat patterns of sentences, words, and strings. Patterns can have many other forms; for example, visual images, acoustic signals, and other physical quantities such as electrical, pressure, temperature, etc., that appear in engineering problems. Patterns can also be abstract without any associated physical meaning. These patterns may be represented in various ways such as digital, analog, and other units. Some of these types of patterns can be associated with combinatorics. There has been extensive research on combinatorial pattern matching (Kapralski, 1993) Computer music can be a specialized application domain of combinatorics of acoustic signals.

**7. Scientific discovery**

For certain types of knowledge discovery problems, generation of combinatorial sequences may become necessary in the process of yielding candidate solutions. For example, in scientific discovery, we may want to have a sequence of plausible chemical/biological reactions and their formations (Itaketo, 2010) In each step of the sequence, we may generate combinatorial sequences of chemical/biological radicals, bases, and molecular compounds as candidate solutions and may select the most likely ones under certain rules and constraints. In another example, certain areas of mathematics, such as graph theory and number theory, may generate combinatorial sequences as candidate solutions.

**8. Databases and data mining**

Queries in databases are multiple join operations that are permutations of the constituent join operations. Determining an optimal permutation that gives minimum cost is a common and important problem (Massini, 2003) Data mining or knowledge discovery in databases is a relatively new field that aims at distilling useful information, often from large databases. In this area, techniques employing symbolic AI can manipulate combinatorial sequences of atoms or information elements. Non-symbolic knowledge discovery techniques, such as genetic algorithms and genetic programming, most commonly deal with solutions in the form of sequences of bits, digits, characters, and sometimes Lisp program elements. Neural networks, another domain of non-symbolic AI, sometimes deal with combinatorial patterns. Knowledge discovery techniques under uncertainty, such as Bayesian networks, Dempster-Shafer theory, fuzzy logic, and rough set theory, may have combinatorial solutions (Nandi, Kar and Pal Chaudhuri, 1994).

**9. Operations research**

Many optimization problems in operations research (OR) involve combinatorics. The job scheduling problem is essentially a sequencing problem to determine the order of jobs to be processed in an effort to minimize the total time, cost, etc. Here, jobs can be in a computer system, network, or processing plant. Many problems involving graphs or networks also deal with the order of vertices and edges. The traveling salesperson problem is to determine the order of cities to be visited to minimize the total distance (Mycielski et al., 1997). The shortest path problem of a graph is to determine a sequence of edges, the total length of which is minimum. Oftentimes, these problems are computationally difficult--e.g., NP-complete or NP-hard--and, therefore, require extensive research.

### 10. Simulation

Permutations and combinations can be employed for simulations in many areas. Permutations representing various genotype-phenotype associations are employed in genetics simulations (Kaufman, Perlman and Speciner, 2003) other areas that employ permutations and combinations for simulations include networks, cryptography, databases and OR.

### 11. Homeland security

This is a very specialized problem domain that has become a major national challenge after 9/11. To confront this challenge, many intelligent computing techniques have been applied, including intelligent pattern analyses of human faces, X-ray images, chemical components, data from a distributed network of wireless sensors, etc. Natural language processing and data mining techniques have been applied to sift through and monitor the tremendous
accumulation of electronic communication data. Since combinatorics are extensively applied to these intelligent computing techniques, there is a wide spectrum of potentials for the national security issue. Some specific examples may include string searching algorithms and their performance analysis in communication data, pre- and postanalysis of combinatorial sequences of information elements, and combinatorial pattern matching.

## Conclusion

From the examples solved above we□ve seen how to solve equations of permutation and combination, on how to determine n number of items and how it can be selected or arranged in r number of ways and also introduced Anekwe□s method of swapping factorials in the case of determining r number of items and through worked example we have justify the validity of Anekwe□s method of swapping factorials.

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